Precautionary saving, wage risk, and cyclical reallocation*

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Abstract

Workers face wage risks associated with job-to-job transitions, which are comparable in magnitude to unemployment risk but largely beyond the reach of traditional social security safety nets. I examine the consequences of cyclical variations in wage risk by introducing a job ladder into the state-of-the-art Heterogeneous Agent New Keynesian model. The dynamic job ladder propagates productivity shocks through two channels: fluctuations in worker reallocation rate between jobs and cyclical variations in relative wages. Adverse productivity shocks lead to an endogenous increase in wage risk, triggering a strong precautionary-saving response that depresses consumption and output. I find that endogenous fluctuations in wage risk amplify recessions by 40% compared to a scenario with no dynamic job ladder, with the predominant share of the amplification arising from changes in reallocation rates.

Keywords: income risk; precautionary saving; on-the-job search.

JEL classification numbers: E21, E24, E32, J64

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1 Introduction

Labor market disruptions during the recent Covid crisis have renewed interest in the role of labor markets in deepening economic recessions. The New-Keynesian perspective suggests that labor market frictions exacerbate recessions by increasing unemployment risk, which drives up precautionary saving and depresses demand (Ravn and Sterk, 2017; Challe, 2020). However, unemployment risk captures only part of the income risks generated by a frictional labor market. About half of all new hires in the U.S. move directly from one job to another, bypassing unemployment (Karahan et al., 2017). This makes income risk stemming from job-to-job transitions—known as wage risk—just as substantial as the risk associated with unemployment. Notably, fluctuations in the job-to-job movements over the business cycle are documented to be more persistent than other types of labor flows, indicating a high degree of wage risk inertia. Moreover, wage risk is immune to traditional safety nets, such as unemployment insurance, which only smooths the impact of unemployment risk. Despite the importance and unique features of wage risk, its role in demand fluctuations has been overlooked, leaving a gap in our understanding of how frictional labor markets shape economic downturns.

This paper investigates the role of uninsured cyclical wage risk in amplifying economic recessions. For this, I employ a state-of-the-art Heterogeneous Agent New-Keynesian model with Search and Matching frictions (HANK&SAM), which I extend by incorporating a job ladder, allowing workers to search for more lucrative employment opportunities while employed. The possibility of reallocation across the job ladder gives rise to wage risk. My contribution is twofold. First, I theoretically identify two novel channels of wage risk transmission: the reallocation channel and the relative wage channel. During recessions, the probability of upward movement on the job ladder declines, giving rise to the reallocation channel, while the decrease in wage gains from moving up the ladder brings about the relative wage channel. Both channels increase wage risk, stimulating precautionary saving and depressing consumption and output. Second, I quantify the impact of wage risk in shaping cyclical demand for precautionary savings within the numerical version of the HANK&SAM model.

My main finding is that cyclical wage risk strongly reinforces recessions – by an additional 40% upon impact under an adverse productivity shock, compared to a model with only unemployment risk. Moreover, wage risk leads to much more persistent responses in consumption and output, generating a deeper and more protracted recession. The amplifying impact of wage risk dynamically surpasses that of unemployment risk, with the predominant share of this effect stemming from the reallocation channel. Notably, this strong amplification results from precautionary asset accumulation by workers at different positions on the job ladder. I show that this contrasts with scenarios of limited heterogeneity where, in line with much of the HANK&SAM literature, the asset distribution collapses to a single mass point, resulting in only a moderate 10% amplification of the recession.

At the core of my model lies the wage negotiation process between the firm and the worker. The wage is determined in the spirit of the bargaining game of Binmore, Rubinstein and Wolinsky (1986) and is also similar to the model of Gottfries (2018), in which the worker and the firm take turns making wage offers. The resulting bargaining wage outcome internalizes the worker turnover rate, including the probability of poaching. As a result, the equilibrium wage in my economy depends on the job-to-job transition rate, which, in turn, also depends on the equilibrium wage. This equilibrium interplay distinguishes my model from other HANK&SAM frameworks with on-the-job search,[1] as labor market disruptions now trigger not only fluctuations in the job-to-job transition rate but also immediate wage adjustments, giving rise to reallocation and relative wage channels, respectively, for the propagation of aggregate shocks.

The reallocation channel arises from the relationship between labor market tightness and the probability of transitioning to a better match. When a negative shock hits the economy, a slack labor market slows the pace of reallocation toward more productive matches, causing workers to climb the job ladder more slowly. The relative wage arises because changes in the turnover rate impact the equilibrium bargained wage. With a decreased probability of workers being poached, firms capture a greater share of the surplus. This leads to wage adjustments primarily for low-productivity matches, but not for high-productivity matches, as they are less vulnerable to poaching due to their position at the top of the productivity distribution. As a result, wages adjust unevenly, creating an additional layer of time-varying wage risk for workers, which can be procyclical or countercyclical depending on their position on the job ladder.

To demonstrate how the two forces unfold, I incorporate these mechanisms into a HANK&SAM model that includes imperfections in both goods and financial markets, along with risk-averse workers who can accumulate liquid wealth to self-insure against idiosyncratic unemployment and wage risks. I begin by considering a limiting case of my model with a zero net asset supply, similar to the approach taken by Ravn and Sterk (2017, 2021) and Challe (2020). I construct an equilibrium wherein the only type of workers not liquidity-constrained are those positioned at the upper bound of the productivity distribution. I show analytically that in this economy, the relative wage and the reallocation channels, operate in opposite directions. On the one hand, workers at the top of the job ladder are less concerned about descending to the bottom because it implies a smaller wage decline than in normal times. This entails a reduction in downward wage risk, leading to decreased precautionary savings and stabilizing the economy. On the other hand, a time-varying job-to-job reallocation rate generates a countercyclical precautionary saving mo-

[1] The works of Moscarini and Postel-Vinay (2017*a*), Faccini and Melosi (2021), Alves (2022), and Birinci et al. (2022) also incorporate on-the-job search within New-Keynesian models in various forms. These models, however, assume zero bargaining power for workers, implying that negotiated wages are unaffected by turnover rates. These studies leverage the renegotiation mechanism driven by outside offers to explain the recent puzzling trend of missing deflation. In contrast, my model focuses on precautionary saving prompted by cyclical wage risk. Here, workers have non-zero bargaining power, leading to cyclical adjustments in wage levels even in the absence of outside offers.

tive, acting as a destabilizing force. In bad times, destabilization through the reallocation channel occurs because, if workers separate, it becomes more difficult for them to recover their position on the job ladder due to slow job-to-job reallocation. As a result, workers' fear of becoming stuck in low-productivity matches implies greater wage risk and prompts precautionary saving.

Analysis of the zero-liquidity equilibrium, however, is restrictive, as it abstracts from the effects of cyclical wage risk on aggregate demand among low-productivity and unemployed workers. To quantify the effect of wage risk in a more realistic setting, I relax the assumption of zero liquidity and evaluate the time-varying precautionary saving motive in an economy with non-degenerate wealth distribution. I calibrate my quantitative model to match the key moments of the US data, including liquidity, labor market flows, and residual wage dispersion. My model can replicate various salient data features, such as the negative skewness and high kurtosis of the income growth distribution, which is a natural outcome of job-to-job movements.

Quantitatively, I show that the effect of aggregate productivity shocks undergoes substantial amplification through both the reallocation and relative wage channels. These two forces increase unemployment and consumption responses by up to 40 percent compared to a counterfactual scenario where these channels are not operative. Importantly, relaxing the assumption of zero liquidity reverses the stabilizing impact of the relative wage channel. In a quantitative model where demand for precautionary savings also arises from workers in low-productivity matches, unequal wage adjustments across the distribution increase wage risk and further boost precautionary savings among low-productive workers. This occurs because, during a recession, workers in low-productivity matches not only expect slower advancement up the job ladder but also experience smaller wage increases associated with any job ladder progress compared to normal times. As a result, the destabilizing impact of relative wage changes at the bottom of the job ladder offsets the stabilizing effect at the top, further depressing consumption and output.

I conclude my analysis by examining the reallocation and relative wage channels from an empirical perspective. Using micro-level data from the Survey of Income and Program Participation (SIPP), I explore the empirical relationship between wage growth and job transitions and contrast it with the theoretical predictions derived from my model. Specifically, the model predicts that wage growth should be positively correlated with job-to-job transition rates for job switchers (reflecting the reallocation channel) and negatively correlated for job stayers (reflecting the relative wage channel). To examine these relationships, I adopt the two-step methodology of Moscarini and Postel-Vinay (2017*b*), which first filters out compositional effects and then estimates the correlation between job transitions and wage growth. My empirical findings support the presence of both the reallocation and relative wage channels.

Related literature. This paper contributes to the burgeoning literature on the role of income risk in the HANK economies. Werning (2015) demonstrates that income risk cyclicality generally shapes the aggregate consumption response to exogenous shocks. Challe and Ragot (2016), Ravn and Sterk (2017) show that endogenous countercyclical unemployment risk coming from labor market imperfections leads to destabilizing demand feedback due to a precautionary saving motive. Ravn and Sterk (2021) derive this result theoretically within a tractable zero-liquidity setup: a combination of debt limits and zero net asset supply ensures degenerate wealth distribution. Challe (2020) and Bonciani and Oh (2021) show that in a zero-liquidity economy, the appropriate monetary policy response almost entirely offsets the destabilizing impact of endogenous unemployment risk. Broer et al. (2021), and Jung (2023) examine the role of endogenous job destruction in precautionary saving feedback. I contribute to this literature by explicitly incorporating cyclical wage risk—alongside unemployment risk—arising from on-the-job search and examining its role in the propagation of aggregate shocks. In my model, the zero-liquidity scenario emerges as a limiting case, and the effect of wage risk on aggregate demand in this context remains generally ambiguous.

Several studies depart from the assumption of zero liquidity and explore the effect of income risk in the economy featuring non-degenerate wealth distribution. Cho (2023) documents that in an economy characterized by a non-degenerate wealth distribution and search-and-matching frictions, income risk plays a limited role due to a strong composition effect. This effect occurs because not only does the probability of unemployment change, but the actual number of unemployed workers also increases. This rise in unemployment has a essential impact on aggregate savings, contrasting the impact of cyclical income risk in the economies where the composition effect is absent either because of limited heterogeneity (Challe et al., 2017) or because changes in income risk are exogenous (Bayer et al., 2019). Graves (2020) shows that a two-asset economy features strong precautionary saving demand feedback even though the economy experiences a change in unemployment-employment composition. In my model households can save only in unique liquid assets, but the shock may exhibit strong amplification from the precautionary channel because the job ladder can revert the composition effect.

The frictional labor market with on-the-job search in my model relies on the assumptions of random search and bilateral bargaining in the spirit of Pissarides, as opposed to the wage posting models. For the wage determination, I use the result of Gottfries (2018), that for the economies with heterogeneous employer-worker matches and infrequent wage renegotiation, equilibrium wage maximizes the Nash product. For the empirical test of the model implications, I employ the methodologies of Faberman, Justiniano et al. (2015); Moscarini and Postel-Vinay (2017*b*); Karahan et al. (2017), which explore the performance of various labor market flows as predictors of individual wage growth.

Finally, several recent works incorporate job ladders in a general equilibrium framework with risk-averse households. Moscarini and Postel-Vinay (2017*a*) show that cyclical movement in employment allocation induced by job-to-job activities can affect inflation dynamics. Faccini and Melosi (2021) use this insight to explain the missing inflation puzzle. Further, Alves (2022) demonstrates that cyclical job reallocation following financial shock can account for the missing disinflation in the quantitative HANK & SaM framework and Birinci et al. (2022) analyzes this environment from the normative point of view. Similarly to these papers, I incorporate a stochastic job ladder in the general equilibrium model with risk-averse households but concentrate on aggregate demand rather than inflation dynamics.

2 Wage adjustment in the model with on-the-job search.

I start by considering a simple economy with a frictional labor market and risk-neutral workers. This section aims to demonstrate how the introduction of on-the-job search influences bargained wages compared to a model without on-the-job search in stationary equilibrium and its cyclical adjustment. I show that the business cycle variation in job-to-job transition rates reshapes workers' job prospects, impacting not only the rate of reallocation to better jobs but also the wage increases associated with any reallocation. Specifically, I demonstrate that the possibility of job-to-job reallocation reduces wages for low-productive matches and makes wages relatively more volatile at the top of productivity distribution. This wage dynamics becomes a source of additional countercyclical income risk in the economy with risk-averse households, which I will develop in Section 3.

I rely on the framework of Gottfries (2018) featuring random search and non-cooperative wage bargaining. Wage determination unfolds in the spirit of the bargaining game of Binmore, Rubinstein and Wolinsky (1986), wherein a firm and a worker alternate in proposing wage contracts. The turnover rate depends on contracted wages and contracted wages are infrequently renegotiated, which allows for overcoming the problem of a non-convex bargaining set (Shimer, 2006) that might be an issue in economies with on-the-job search (Pissarides, 1994; Mortensen, 2003).

2.1 Environment

Time is continuous and runs indefinitely.

The economy consists of workers and firms. Workers are ex-ante identical infinitely-living risk-neutral agents. Each worker can be in one of two states: employed or unemployed. When unemployed, workers are looking for a job. During the search, unemployed workers receive a flow of real return b, which can be interpreted as home production or unemployment insurance benefits. When employed, workers are matched with firms and earn match-specific wages $w(x)$, where x is the idiosyncratic productivity of a match. Besides, employed workers always search for a better job, but with lower intensity than unemployed.

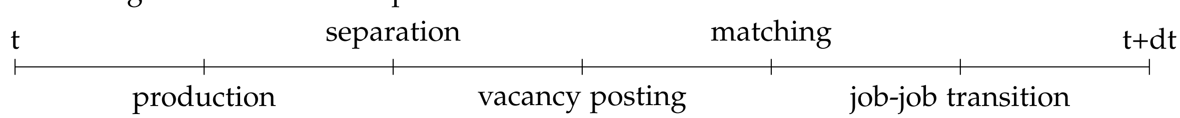
Ex-ante identical risk-neutral firms are on the other side of the labor market. Each firm has one job, which can be either filled or vacant. Firms with a vacant job search for an employee by posting vacancies at a per-period real cost of c. Firms with filled jobs produce homogeneous consumption goods with linear technology using only labor as input. If a firm neither produces nor searches, its job is destroyed.

The productivity of each firm consists of idiosyncratic component x, which is match-specific, and aggregate component z, which is common for all firms in the economy. The worker-firm separation occurs for two reasons. First, an idiosyncratic adverse shock hits a firm with Poisson intensity λ and destroys the match. In this scenario, a worker engaged in the match becomes unemployed. The second possibility for job destruction is poaching. Separation occurs if the worker can form a better match with an outside firm offering a job. In this case, workers change their jobs bypassing the unemployment state, in contrast to the separation after an adverse shock. However, from the firm's point of view, separation induced by shock and poaching are equivalent.

I denote the total population of workers in the economy as L. The corresponding number of unemployed workers and vacant jobs are uL and vL, respectively. The effective number of job seekers is $uL + s(1-u)L$, where $1 \geq s \geq 0$ refers to the search efficiency of employed workers, which generally can differ from search intensity of unemployed. To ensure analytical tractability, I assume $s = 1$ throughout this section.

The number of job matches is given by homogeneous of degree one matching function $m(.,.)$, such that $mL = m(vL, (u + s(1-u))L)$. Defining labor market tightness as $\theta = \frac{v}{u+s(1-u)}$, the rate at which workers arrive to vacancies is $\frac{mL}{vL} = m(1, \frac{1}{\theta}) \equiv q(\theta)$. Workers' corresponding job arrival rate is $\theta q(\theta)$.

The timing of events within a period is as follows:



1. Firms produce consumption good according to their productivity px and pay to workers wage $w(x)$. Unemployed households enjoy benefits b.

2. Adverse separation shocks hit the existing matches with exogenous rate λ.

3. Unemployed and employed workers search for jobs, firms post vacancies, and random matches occur.

4. When a worker meets with a firm, they draw random productivity value from the known distribution $G(x)$, with $G(\overline{x}) = 0$ and $G(\underline{x}) = 1$. Given the productivity of the match x, the firm and potential employee bargain for the wage.

5. Workers decide whether to accept the job with wage $w(x)$.

2.2 Equilibrium wage

Wage contracts do not last forever, as in Gottfries (2018). Instead, wages are infrequently renegotiated with an exogenous intensity independent of the offer arrival rate. Throughout the paper, I assume that the frequency of renegotiation approaches infinity, and I discuss this assumption later.

When a firm and a worker are matched, they negotiate wages via the strategic bargaining game of Binmore, Rubinstein and Wolinsky (1986). Players take turns making offers in a bargaining game, with the firm initiating the first offer. Following each offer, the responder decides whether to accept or reject it. Accepted offers result in the agreed-upon wage payoffs for both parties, while rejected offers carry a probability of leading to a breakdown in the bargaining process. After the worker extends an offer, the probability of no breakdown is $(1-d\tilde{t})^{\phi}$, while it is $(1-d\tilde{t})^{1-\phi}$ following an offer from the firm. Following Shimer (2006), I assume that bargaining takes place in artificial time $\tilde{t}$ so that the probability of breakdown remains the only relevant friction. In the event of a breakdown, the worker transits to unemployment, and the firm exits without any additional value.

The key insight of Gottfries (2018) is that in an on-the-job search model with bargaining, infrequent renegotiation of wages, and match-specific productivities, the equilibrium wage maximizes the Nash product even though the bargaining set might be non-convex. I use this result to derive an equilibrium wage in my economy. Denoting worker surplus as S^w and firm surplus as S^f, the bargaining game has a unique solution, such that $w = \arg\max_w (S^w)^{\phi}(S^f)^{1-\phi}$. The following proposition establishes wages as a result of the bargaining game

Proposition 1. *The wage resulting from the bargaining game is given by*

$$w(x) = (1-\phi)b + \phi z(x + \theta c\Theta) - \underbrace{\phi f^{ee} \int_x^{\overline{x}} J(s)dG(s)}_{\Delta(x)} \tag{1}$$

where J(x) is a value of a firm with productivity x, $f^{ee} = s\theta q(\theta)$ *and* $\Theta = \frac{\int_{\underline{x}}^{\overline{x}} J(s)dG(s)}{\int_{\underline{x}}^{\overline{x}} F(s)J(s)dG(s)}$. *See proof in Appendix A*

Consider the first two terms in Equation (1). Wage is growing in benefit b since high unemployment benefit increases the outside value of the worker. Besides, workers receive part of the output zx, and the firm's opportunity costs $z\theta c$, proportional to the bargaining power ϕ. These two terms constitute a standard wage expression in models *without* on-the-job search as, for example, in Mortensen and Pissarides (1994) (without on-the-job search $\Theta = 1$).

Additional term $\Delta(x)$ in Equation (1) differs wage expression in this economy from the setups that either abstract from job-job reallocations or incorporate it in the more stylized form as in

Pissarides (1994). Term $\Delta(x)$ corresponds to the compensation for the expected poaching of a worker. When job-to-job transitions occur, workers climb the job ladder and experience an increase in utility (value) stemming from higher wages, contrasting with firms, for whom the loss of an employee (due to poaching) implies job destruction. Thus, at a given wage, the introduction of on-the-job search impacts the value of workers and firms in opposite directions: it increases the value of workers while decreasing the value of firms. Since the surplus sharing condition $\phi S_f = (1-\phi)S_w$ must hold, the new equilibrium wage mechanically decreases, as it should account for the expected job-to-job transitions of workers. Generally, it implies that the negotiated wage is lower than in the baseline result of Mortensen and Pissarides (1994).

From Equation (1), it is clear that $\Delta(x)$ is necessarily non-negative and increasing in labor-market tightness θ. This property is not surprising since high labor-market tightness increases the job arrival rate and boosts the probability of poaching. Another essential feature of $\Delta(x)$ is that it is decreasing in x, with $\Delta(\overline{x}) = 0$. Poaching is more likely when a match is low-productive, while poaching is impossible at the upper bound of productivity distribution $\overline{x}$.

Equation (1) provides a wage expression, albeit not in closed form, as the firm value $J(x)$ is an endogenous object. Additionally, the exact form of $\Delta(x)$ depends on the productivity distribution $G(x)$. Fortunately, for certain distributions $G(x)$, the term $\Delta(x)$ can be expressed analytically. Proposition 1 establishes equilibrium wage in closed form for the uniform productivity distribution.

Proposition 2. *Assume that $G(x)$ is a continuous uniform distribution with support $x \in [\overline{x}, \underline{x}]$. In this economy, the wage schedule is given by*

$$w(x) = (1-\phi)b + \phi z(x + \theta c \Theta) - \underbrace{\phi z(\bar{x} - x) - \bar{x} z \left[\frac{C(x)^{\phi} - C(\bar{x})^{\phi}}{C(\underline{x})^{\phi-1}} \right]}_{\Delta(x)} \tag{2}$$

where $C(x) = \frac{r+\lambda}{\theta q(\theta)} + 1 - \frac{x}{\bar{x}}$. See proof in Appendix A

To illustrate the effect of on-job search on wage function, I numerically find equilibrium in the model[2] using calibration that matches conventional moments in the data.[3] Figure 1, plots equilibrium wage as a function of productivity match x.

At the lower support of the productivity distribution equilibrium wage in the model with on-the-job search is reduced compared to the models where employees do not search. When idiosyncratic productivity is small, the probability of poaching is high, translating into a larger

[2]Equilibrium to this economy is defined by wage $w(x)$, labor-market tightness θ, and firm distribution $F(x)$ which are consistent with the wage equation, and the job creation condition

[3]The model is in monthly rates. The separation rate is $\lambda = 0.034$, search intensity ensures that the job-finding rate $\theta q(\theta) = 0.45$. Workers' bargaining power $\phi = 0.5$. The Hosios efficiency condition is satisfied. The vacancy posting costs c are those that match the labor-market tightness $\theta = 1$. Productivity $G(x) \sim U[0.9, 1]$.

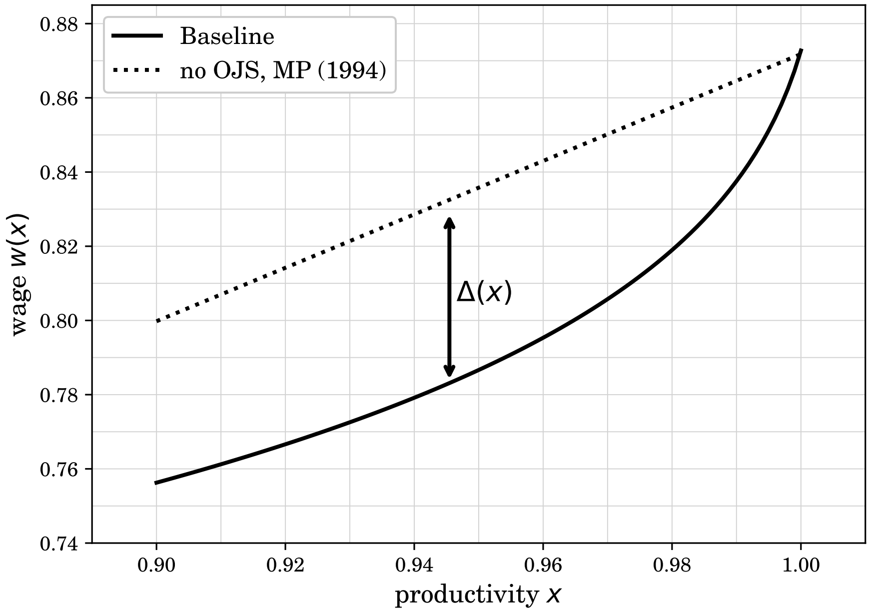


Figure 1: Wage schedule in the model with on-job search

The solid line represents wage as a function of idiosyncratic productivity x in the model with on-the-job search laid out in this section. The dotted line is the wage equation as in Mortensen and Pissarides (1994) where the on-the-job search is absent.

compensation term $\Delta(x)$. It means that, as a consequence, workers and firms agree to set wages at a lower level through negotiation. When idiosyncratic productivity is high, there is a lower chance that a firm with random productivity lures away workers, therefore term $\Delta(x)$ shrinks and the negotiated wage is higher. The resulting wage schedule is much steeper compared to the case with no on-the-job search, implying that the transition from low-productive to high-productive matches is generally associated with larger increases in wage than in Mortensen and Pissarides (1994).

2.3 Cyclical wage adjustment

Next, I examine how the wage adjusts at different parts of the productivity distribution following a change in aggregate productivity. To this end, I consider a permanent decrease in productivity component z that is common for all matches in the economy.

A negative productivity shock has a conventional effect on the economy. Low productivity decreases the number of new vacancies, the labor market tightness, and ultimately job-finding and job-job transition rates. With lower productivity and a weaker labor market, workers renegotiate wages. Formally, a change in wages could be expressed as:

$$\frac{dw(x)}{dz} = \phi(x + \theta c\Theta) + \phi zc\Theta\frac{d\theta}{dz} - d\frac{\phi f^{ee}\int_x^{\overline{x}} J(s)dG(s)}{dz} \tag{3}$$

The first and the second terms in Equation (3) relate the change in wage with the direct effect of productivity (direct channel) and with the change in labor market tightness (opportunity costs channel). The main object of interest is the third term in Equation (3), which enters the equation with the negative sign. When the labor market is slack, the contact rate is low, and the probability of poaching declines, which shrinks the $\Delta(x)$ term. Graphically, Figure 2 illustrates the overall impact of lower productivity on equilibrium wage.

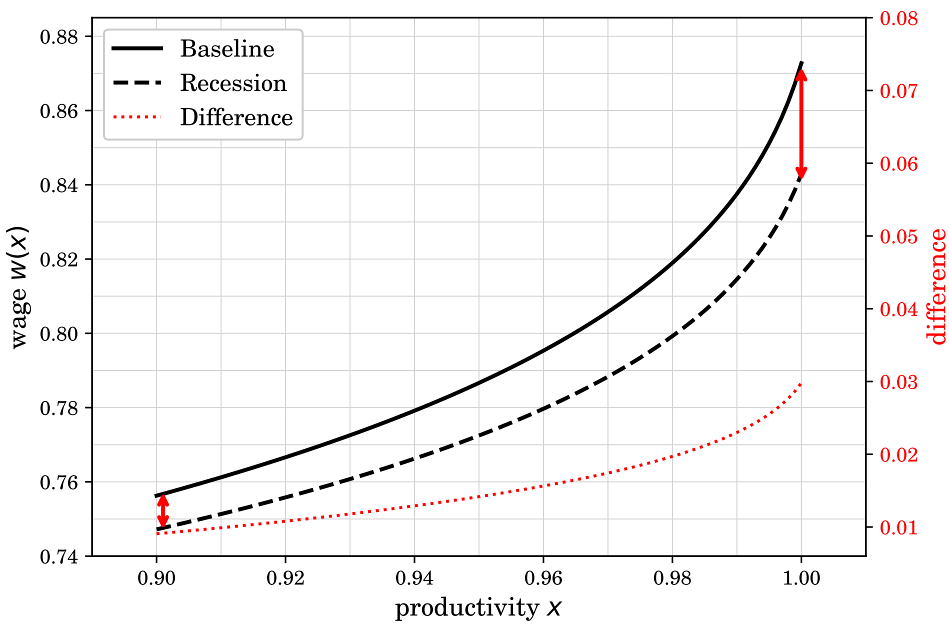


Figure 2: Wage schedule in the model with on-job search

Black curves plot the steady-state wage functions in the model with on-the-job search before and after a permanent decrease in aggregate productivity z. The red line shows the difference between the two wage functions

As the dashed line indicates, a decline in aggregate productivity shifts wages for every idiosyncratic productivity level x. The change in wages originates from the (i) direct, (ii) opportunity costs, and (iii) poaching channels. However, the shift is not parallel: at the top of the productivity distribution, bargained wage drops dramatically, while for low x wages experience only a moderate decline. This asymmetry comes from the poaching channel; when the labor market is slack, the $\Delta(x)$ term declines, which has a disproportionately strong positive impact on wages at the bottom of the productivity distribution. As a result, a fall in productivity makes the wage differentials between more productive and less productive jobs smaller.

2.4 Discussion

This section highlights the role of on-job search in cyclical wage adjustment. Workers extract part of the total surplus because of their bargaining power. The size of the total surplus depends on the contact rate, and ultimately, the probability of poaching. When aggregate conditions change, wages also change through conventional direct and opportunity cost channels as well as through the changes in poaching probability. The latter impacts wages at the bottom of the productivity distribution, as workers in low-productivity matches are subject to poaching.

The central result of this section is that the wage differential between high-productivity and low-productivity matches decreases during recessions. This wage characteristic becomes crucial in Section 3, where I deviate from the assumption of linear utility and investigate the role of precautionary saving. When the labor market is weak, workers in low-productivity matches not only face a low probability of reallocating to a more productive job but also experience a smaller wage increase with each reallocation due to smaller wage differentials across different positions on the job ladder, compared to normal times.

Importantly, my model differs from the classical framework of Pissarides (1994), which assumes that new matches are always at the upper bound of the productivity distribution. As a result, workers reallocating to a new employer always transit to the very top of the job ladder, the turnover rate doesn't depend on the bargained wage and the poaching channel is absent.

Finally, the main result on wage adjustment is robust to the incorporation of stochastic idiosyncratic productivity and endogenous destruction in the spirit of Mortensen and Pissarides (1994); Pissarides (1994). As I show in Appendix A, both extensions do not change the wage equation. Moreover, the model can be extended to include reallocation shocks or search costs, with the same qualitative results.

3 Quantitative model

In this section, I integrate the key theoretical mechanism from Section 2 into a conventional HANK & SAM model, in the spirit of Ravn and Sterk (2017, 2021) and Challe (2020). Like before, there exists a frictional labor market. However, households are now assumed to be risk-averse, responding to changes in income risk through precautionary saving in the absence of other means of insurance. Additionally, the model includes imperfections in the goods market, which, together with financial and labor market frictions, give rise to powerful amplification mechanisms in general equilibrium.

3.1 The model

Households. The economy is populated by two types of households: workers and firm owners. There is a measure-one continuum of ex-ante identical risk-averse workers participating in the labor market and a measure Ω of risk-neutral firm owners that run all firms in the economy, collect profits, and supply no labor.

Workers can be employed or unemployed. Unemployed workers earn exogenous home production income b_t and search for a job. Employed workers supply one unit of labor inelastically and can be in a match with a firm with one of the productivities x_1, x_2, ... x_n with corresponding wages $w_t(x_{it})$. Besides, employed workers always search for a better job with search intensity s.

Workers maximize lifetime value $W(.)$ by choosing a stream of consumption c_t subject to a budget constraint:

$$\begin{aligned} W(a_{i0}, x_{i0}) &= \max_{\{c_{it}\}_{t\geq 0}} E_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt \\ \dot{a}_{it} &= r_t^r \cdot a_{it} + w_t(x_{it}) - c_{it} \\ x_{it} &\in \{x_1, x_2, \dots x_n, 0\} \end{aligned} \tag{4}$$

where $u(.)$ is a utility function with constant relative risk aversion σ; workers discount the future at rate ρ and save in a liquid asset a_t at a riskless real rate of return r_t^r. Here, I use w_t to denote generic labor income coming from either the labor market or home production, such that $w_t(x_t) = b_t$ if $x_t = 0$.

Aside from the budget constraint, workers face exogenous debt limit $\underline{a}$:

$$a_t \geq \underline{a}(x_t) \tag{5}$$

I follow Ravn and Sterk (2021) by assuming that the tightness of the borrowing constraint depends on the labor income. For unemployed workers, the debt limit is zero, such that $\underline{a}(0) = 0$.

Firms. The production structure of the economy incorporates three vertically integrated sectors: labor intermediaries, retailers, and final goods producers. Labor intermediaries hire workers from the frictional labor market and produce labor services, which they sell in a competitive market. Monopolistic retailers are in the middle of the supply chain. They purchase labor services to produce specialized intermediate goods, which they supply to final goods firms. The final goods firms purchase these specialized intermediate goods and aggregate them into homogeneous consumption goods.

Final goods firms. The representative firm in the final goods sector employs $Y_{j,t}$ units of each specialized good $j \in [0,1]$, each priced at $P_{j,t}$, to produce Y_t units of the consumption good. The production technology exhibits constant returns to scale, with ϵ representing the elasticity of demand for each intermediate good:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{6}$$

The optimal input combination chosen by the final goods firm yields the demand function for each intermediate good j as follows:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} Y_t \tag{7}$$

In equilibrium, competition drives the profits of final goods firms to zero, determining the price index P_t associated with Equation (7) as $P_t = \left(\int_0^1 P_{j,t}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$.

Retail firms. The retail sector is a middle production layer of the economy. A retail firm purchases labor services from intermediaries to produce a specialized good j according to the following technology:

$$Y_{j,t} = A_t L_{j,t}^{1-\alpha} \tag{8}$$

where $L_{j,t}$ denotes the amount of labor services purchased by the retail firm producing good j, $\alpha \in [0,1]$ is a parameter, and A_t is the aggregate productivity common to all retail firms. The retail firm has monopolistic power and sets the price of its goods to maximize the flow of expected discounted future profits, while facing quadratic costs of price adjustments:

$$\max_{\{P_{j,t}\}_{t \geq 0}} \int_0^\infty e^{-\int_0^t r(s)ds} \left\{ \left(P_{j,t} - mc_{j,t} \right) Y_{j,t} - \frac{\chi}{2} \left(\frac{\dot{P}_{j,t}}{P_{j,t}} \right)^2 Y_t \right\} dt \tag{9}$$

subject to the demand function (7) and the production function (8). Here, $\chi > 0$ measures the degree of nominal rigidity in price adjustments, and $mc_{j,t}$ denotes the retailer's marginal costs.

Labor intermediaries. Labor intermediaries hire workers in the frictional labor market, convert this labor into homogeneous labor services, and sell these services to retail firms in a competitive market. The problem faced by a labor intermediary is similar to those described in Section 2. To repeat, all labor intermediary firms are identical ex-ante and can post vacancies at a real cost of c^f. Upon meeting a worker, the firm (and potential worker) draws idiosyncratic productivity x from the productivity distribution $G(x)$. If a new match is formed, the wage $w_t(x)$ is determined through bilateral bargaining.

Labor intermediary firms with filled jobs produce labor services using linear technology and sell these services at a real price of p_t^I. Matches can break for two reasons: either due to an exogenous idiosyncratic separation shock with Poisson intensity λ, or through poaching, which occurs with an endogenous probability $f_t^{ee}(1 - G(x))$. Here, f_t^{ee} represents the contact rate of an employed worker with outside firms, and $1 - G(x)$ is the probability that the productivity of a match with a random outside firm will be higher than that in the current match.

The job-finding rate f_t, the contact rate f_t^{ee}, and the job-filling rate q_t are functions of labor market tightness θ and are defined as follows:

$$f_t(\theta) = m\theta_t^{1-\eta}, \qquad f_t^{ee}(\theta) = sm\theta_t^{1-\eta}, \qquad q_t(\theta) = m\theta_t^{\eta} \tag{10}$$

where η is the matching elasticity, and m is a scaling parameter. As described in the analytical section, labor market tightness is defined as $\theta_t = \frac{v_t}{u_t + s(1-u_t)}$, where v_t denotes the number of vacancies and u_t is the unemployment rate. The free-entry condition determines the number of posted vacancies, while the unemployment rate evolves according to the recursion:

$$\dot{u}_t = \lambda(1 - u_t) - f_t u_t \tag{11}$$

Central Bank and Government. The Central Bank sets the nominal interest rate in line with the Taylor rule.

$$i_t = r^{ss} + \phi_\pi \pi_t \tag{12}$$

The government runs public debt D by issuing one-period risk-less bonds. To finance debt costs the government imposes a lump-sum tax on firm owners. The government budget constraint is

$$\dot{D}_t = r_t \cdot D_t - \tau_t \tag{13}$$

where τ_t is the total amount of taxes levied by the government. I assume that the bonds issued by the government and those issued by the households are perfect substitutes and traded on the same market.

3.2 Recursive formulation and wage equation

Next, I present the problem of workers and intermediary firms in the recursive form. For brevity, I replace the terms in brackets corresponding to state variables with indices. The Hamilton-Jacobi-Bellman (HJB) equations for unemployed and employed workers are as follows:

$$\rho W_{a,0,t} = \max_{\{c\}} \left\{ u(c) + \partial_a W_{a,0,t} \left[r_t^r \cdot a + b_t - c \right] + \partial_t W_{a,0,t} + f_t \int_{\underline{x}}^{\overline{x}} g(x) \left[W_{a,x,t} - W_{a,0,t} \right] dx \right\} \tag{14}$$

$$\begin{aligned} \rho W_{a,x,t} = \max_{\{c\}} \Big\{ & u(c) + \partial_a W_{a,x,t} \left[r_t^r \cdot a + w_{a,x,t} - c \right] + f_t^{ee} \int_{x}^{\overline{x}} g(x') \left[W_{a,x',t} - W_{a,x,t} \right] dx' \\ & + \lambda \left[W_{a,0,t} - W_{a,x,t} \right] + \partial_t W_{a,x,t} \Big\} \end{aligned} \tag{15}$$

where the last term in the first line of Equation (15) represents the increase in value associated with the probability of finding a better job through on-the-job search. Both HJB equations are also coupled with the corresponding state-constraint boundary condition $\partial_t W_{\underline{a},x,t} \geq u'(r_t^r \cdot \underline{a} + w_{\underline{a},x,t})$, which ensures that workers remain within the borrowing limit $\underline{a}$.

A pair of HJB equations for firms with vacant and filled jobs are:

$$r_t V_t = -c^f + q_t \int_{\underline{x}}^{\overline{x}} g(x) \left[J_{a,x,t} - V_t \right] dx + \partial_t V_t \tag{16}$$

$$\begin{aligned} r_t J_{a,x,t} = p_t^I x - w_{x,t} + \partial_a J_{a,x,t} \left[r_t \cdot a + w_{x,t} - c \right] + f_t^{ee} \int_x^{\overline{x}} g(x') \left[V_t - J_{a,x,t} \right] dx' \\ + \lambda \left[V_t - J_{a,x,t} \right] + \partial_t J_{a,x,t} \end{aligned} \tag{17}$$

The last term in the first line of Equation (17) indicates that, when poaching occurs, job destruction takes place, causing the value of a firm with a filled job to decrease to that of a firm with a vacant job. The free-entry condition establishes that the value of firms with vacant jobs is zero, i.e., in equilibrium $V_t = 0$ at all times t.

Given Equations (14)-(17), I can derive an expression for the wage that satisfies the surplus-sharing rule given by $(1-\phi)\left[W_{a,x} - W_{a,0} \right] = \phi \left[J_{a,x} - V \right]$ for $x_t \in [\underline{x}, \overline{x}]$. This expression is referred to as the notional wage $w_{a,x}^N$:

$$\begin{aligned} w_{a,x,ss}^N = \phi \frac{p^I x + \partial_a J_{a,x} \left[r \cdot a - c^* \right]}{M_x^W M_x^F} - (1-\phi) \frac{u(c^*) + \partial_a W_{a,x} \left[r \cdot a - c^* \right] - r W_{a,0}}{M_{a,x}^W M_x^C} - \\ - \underbrace{\frac{1}{M_{a,x}^W M_x^C} \phi f^{ee} \int_x^{\overline{x}} J_{a,x'} dG(x')}_{\Delta(x)} \end{aligned} \tag{18}$$

where M_x^C and M_x^F denote the effective discount rates for employed workers and producing firms, respectively. They are defined as $M_x^C = \rho + \lambda + f^{ee} \int_x^{\overline{x}} g(x') dx'$ and $M_x^F = r + \lambda + f^{ee} \int_x^{\overline{x}} g(x') dx'$. Further, $M_{a,x}^W = (1-\phi) \frac{\partial_a W_{a,x}}{M_x^C} + \phi \frac{1 - \partial_a J_{a,x}}{M_x^F}$.

Equation (18) presents the equilibrium equation for wages in the steady state. Appendix B provides both the detailed derivations and the corresponding equilibrium equation outside of the steady state. Two remarks regarding the wage equation are in order.

Firstly, even in a general equilibrium model with multiple production layers and risk-averse workers, the wage equation can still be decoupled into the first two "standard" terms on the right-hand side of Equation (18) and an extra $\Delta(x)$ term. This $\Delta(x)$ term, which compensates for potential poaching, is conceptually and algebraically similar to the result presented in Section 2. Like in the analytical model, $\Delta(x)$ increases with the contact rate f^{ee} and decreases with x, reflecting that poaching is more likely when the match is less productive and the contact rate with outside firms is higher.

Secondly, to accurately capture the dynamics between unemployment and wages, I introduce a degree of wage inertia, η^w. This inertia means that the equilibrium wage is a weighted average of the notional wage w_t^N and the steady-state wage w_{ss}:

$$w_{a,x,t} = (w_{a,x,ss})^{\eta^w} (w^N_{a,x,t})^{(1-\eta^w)} \tag{19}$$

3.3 Equilibrium and solution

Equilibrium in this economy consists of sequences of prices, household policy rules, firm policy rules, job-market transition rates, inflation, taxes, and the distribution of households such that, given the initial distribution:

1. Given aggregate variables, households' and firms' policy rules maximize their corresponding objective functions.
2. Taxes ensure that the government budget constraint holds.
3. The free-entry condition holds.
4. Distribution is consistent with policy rules and aggregate variables.
5. Goods, labor, and asset markets clear. The corresponding market-clearing conditions are:

$$\begin{aligned} Y_t =& C_t + \Theta^a_t + cv_t \\ N_t =& \sum_x \int_a x\Phi(a,x,t)da \\ D_t =& \sum_x \int_a a\Phi(a,x,t)da \end{aligned} \tag{20}$$

To derive policy functions, I reformulate household and firm problems as a system of partial differential equations known as Hamilton-Jacobi-Bellman (HJB) equations. Following Achdou et al. (2022), I discretize the system using the upwind scheme method and solve it using the finite-difference method for derivative approximation. I also formulate a system of corresponding Kolmogorov Forward (KF) equations to track the joint evolution of income and wealth distribution, given households' optimal policy.

4 Limiting case: zero-liquidity economy

Before presenting the quantitative results derived from the calibrated version of the model, I first examine a tractable case of my economy. Specifically, I consider a limiting no-trade equilibrium, which eliminates all distributional effects and allows for an analytical exploration of the aggregate impact of cyclical fluctuations in (i) relative wages and (ii) job-to-job transition rates.

To this end, I assume that the productivity distribution consists of only two states: high (x^h) and low (x^l), with equilibrium wages denoted by w_t^h and w_t^l, respectively. Additionally, I assume that unemployed workers and those in low-productivity matches have limited access to the asset market and cannot issue private bonds. Finally, I consider a scenario in which the government does not provide liquidity.[4]

It can be shown that an equilibrium exists in this economy where tight borrowing constraints ensure that unemployed workers and those in the low-productivity state endogenously become hand-to-mouth agents. For these workers, the Euler equation holds with inequality: they would borrow against future income but are unable to do so. Conversely, workers in the high-productivity state are not hand-to-mouth, and the Euler equation holds with equality for them. Although workers in the high-productivity state follow the Euler equation, in equilibrium, the interest rate adjusts such that they neither save nor borrow, as the government does not provide liquidity and other agents do not participate in the asset market. In this way, the combination of borrowing constraints and zero liquidity sustains a no-trade equilibrium and eliminates cross-sectional wealth heterogeneity. In this context, any movements in the real interest rate will reflect changes in the desired savings of high-productivity workers.

Next, I examine the Euler equation of workers in high-productivity matches, where both precautionary saving and consumption smoothing motives remain operative:

$$\frac{r_t^r - \rho}{\sigma} = \overbrace{\frac{\dot{w}_t^h}{w_t^h}}^{\text{intertemporal substitution}} - \underbrace{\lambda(1-f_t)\frac{w_t^h - b_t}{w_t^h}}_{\text{unemployment precautionary}} - \underbrace{\lambda\frac{f_t^l}{2}\left((1-\frac{f_t^{ee}}{2}) + (1-\frac{f_t}{2})\right)\frac{w_t^h - w_t^l}{w_t^h}}_{\text{wage risk precautionary}} \tag{21}$$

where f_t is the total job-finding rate and f_t^l is the rate of finding a low-productivity job.

Equation (21) is the continuous-time counterpart of the equilibrium condition found in Challe (2020), as well as Ravn and Sterk (2021), extended to accommodate two productivity states. It demonstrates that the change in desired savings and, consequently, the interest rate response in this economy is driven by two fundamental forces: consumption smoothing and precautionary saving.

The consumption smoothing motive arises from the worker's aversion to intertemporal substitution, with the first term on the right-hand side of Equation (21) capturing this effect. The other two terms governing the household's optimal policy are broadly associated with precautionary saving,[5] including terms related to unemployment and wage precautionary behavior. The second term reflects the impact of unemployment risk and the associated cuts in consumption level from w_t^h to b_t on precautionary saving. The last term of Equation (21) introduces a precautionary term

[4] To obtain analytical results, I also adjust the time convention to align with conventional HANK&SAM models with zero liquidity. See Appendix C for details.

[5] I interpret precautionary saving as any saving resulting from uncertainty about future income.

stemming from the wage risk, which, as far as I am aware, has not been previously discussed in the literature. If the job ladder includes at least two productivity states, it has a dual impact on precautionary saving: (i) through the employment-to-employment transition rate f^{ee}, and (ii) through the wage differential $w_t^h - w_t^l$. Note, when $f_t^l = 0$, Equation (21) nests a conventional zero-liquidity HANK&SAM result as a special case.

To examine the dynamic impact of the job ladder, as well as other forces, consider the local dynamics of a zero-liquidity economy around the steady state, as commonly explored in the New Keynesian literature.[6] To denote percentage deviations from the steady state, I use a tilde, while absolute deviations are indicated by a hat sign. The first-order approximation of Equation (21) can be expressed as:

$$\hat{r}_t = \sigma \tilde{w}_t^h + \psi_1 \hat{f}_t + \underbrace{\psi_2 (1 - \frac{w^l}{w^h}) \hat{f}_t^{ee}}_{\text{reallocation}} + \underbrace{\psi_3 \frac{w^l}{w^h} (\tilde{w}_t^l - \tilde{w}_t^h)}_{\text{relative wage}} \tag{22}$$

where $\psi_1 \hat{f}_t < 0$, $\psi_2 = \frac{1}{2}\sigma\lambda\frac{f^l}{2}$, $\psi_3 = \sigma\lambda\frac{f^l}{2}(1 - \frac{f^{ee}}{2})$ and variables without time subscript are the steady-state values.

The interest rate response is determined by fluctuations in wages and labor market transition rates. The former reflects the desire for consumption smoothing due to aversion to intertemporal substitution, while the latter represents the impact of the precautionary saving motive. Consumption-smoothing has a clear stabilizing effect on the economy. Following a negative productivity shock, workers seek to finance current consumption with future labor income as they expect it to grow in the next periods when the effect of the shock vanishes. Given workers' lack of insurance against income risk, the interest rate is further influenced by endogenous fluctuations in unemployment risk. With procyclical labor market tightness, adverse aggregate shocks depress job-finding, aggregate demand, and the real rate of interest. The second term in Equation (22) captures this destabilizing effect of unemployment risk, extensively discussed in theoretical HANK & SAM literature.

In contrast to the baseline HANK & SAM model, my equilibrium Equation (22) introduces precautionary terms associated with wage risk, which, as far as I am aware, has not been previously discussed in the literature. It demonstrates that the job ladder propagates aggregate shocks through two channels, namely the reallocation and relative wage channels.

The *reallocation channel* links savings to the rate of job-to-job transitions. When the job-to-job transition rate is low, the impact of a separation shock becomes pronounced as workers have a reduced probability of returning to a high-productivity state. Consequently, from the workers' perspective, a weak labor market increases the risk of being stuck in low-productivity matches.

[6] I consider a steady state with positive vacancy posting, away from the liquidity trap. Ravn and Sterk (2021) demonstrate that, generally, HANK and SAM models can also have steady states associated with a liquidity trap and an unemployment trap.

This risk prompts workers to increase precautionary savings further, which weakens aggregate demand and exerts a destabilizing influence on the economy. The *relative wage channel*, on the contrary, stabilizes the economy, as the difference between w_t^h and w_t^l tends to diminish, as discussed in Section 2. Intuitively, during a recession, being stuck in a low-productivity match is less risky because the associated decline in wages (and consumption) is smaller than in normal times. The reduction in wage disparities effectively mitigates the wage risk for high-productivity workers, leading to a decrease in precautionary savings.

Generally, the impact of wage risk on aggregate demand remains ambiguous, as it is uncertain whether the reallocation or relative wage channels dominate or counterbalance each other. Moreover, the assumption of zero liquidity in this context may be misleading, as it overlooks the cyclical precautionary saving behavior of low-productivity workers. Notably, at the bottom of the job ladder, the influence of the relative wage channel undergoes a reversal, diminishing upward wage risk and potentially destabilizing the economy. Thus, to explore the impact of wage risk within a more realistic framework, I turn to quantitative analysis.

5 Quantitative Results

In this section, and in all sections that follow, I relax the simplifying assumption of having only two productivity states and instead consider many productivity states. Additionally, I relax the assumption that the government does not provide liquidity. In an economy with positive liquidity, both employed workers across various productivity levels and unemployed workers can save in a unique liquid asset. Different histories of income risk realizations lead to ex-post heterogeneity, resulting in a non-degenerate cross-sectional distribution of wealth. As a result, the strength of the precautionary saving motive becomes idiosyncratic, depending on individual wealth levels and labor earnings.

5.1 Calibration

I calibrate the model to match key features of the U.S. data. The model is in monthly rates. I chose the separation rate λ equals 0.034, which corresponds to the monthly separation rate according to Shimer (2005). I set the search elasticity of matching η and worker's bargaining power ϕ to 0.5, as in Mortensen and Pissarides (1994). To calibrate the productivity density function $g(x)$ and subsequently demonstrate its implications (see Section 3.3), I adopt the approach outlined by Alves (2022), which involves matching moments of residual wage dispersion. As the empirical target, I chose the logarithmic differences between the 90th and 50th percentiles (0.55) and between the 50th and 10th percentiles (0.60) of the residual wage distribution (Autor, Katz and Kearney, 2008).

Parameter	Meaning	Targeted Moment	Target
Labor Market Flows			
λ	Separation rate	Separation probability	0.034
m	Matching efficiency	Job-finding probability	0.45
s	Job-job transition rate	Job-job transition probability	0.025
c	Posting costs	Labor market tightness	1
Income Process			
g	Productivity grid	Wage dispersion (p50/p10, p90/p50)	0.60, 0.55
b	Unemployed benefit	Unemployed consumption decline	15%
New-Keynesian Part			
ϵ	Elasticity of substitution	Aggregate markup	20%
Ξ	Price adjustment costs	Frequency of price adjustment	12 months
ϕ_π	Taylor rule coefficient	Standard	1.2
Liquidity			
ρ	Time preference	Interest rate	2%
$\underline{a}$	Borrowing constraint	Fraction with negative assets	0.15
D	Public debt	Debt-output ratio	1.0
Microfounded			
σ	Relative risk aversion	Standard	2
η	Search elasticity of matching	Standard	0.5
ϕ	Worker's bargaining power	Standard	0.5

Table 1: Parameters and Targets

I calibrate parameters of matching efficiency, search intensity of employed workers, and cost of vacancy posting to match the job-finding rate, job-to-job transition rate, and labor market tightness. Matching efficiency ensures that unemployed workers find a job with a monthly rate of 0.45. The relative search intensity of employed workers s is calibrated to fit the average job-to-job transition rate of 0.025 as documented by Fujita, Moscarini and Postel-Vinay (2020). Finally, the vacancy posting cost c guarantees that the labor-market tightness θ is 1 (Krusell, Mukoyama and Şahin, 2010).

The relative risk aversion is set to 2 for my baseline calibration. The time discounting parameter ρ matches the interest rate of 2% (in annual terms). I chose the size of home production b to match a 15% average consumption decline associated with the transition to unemployment. Wage persistence η^w is calibrated to 0.9. The elasticity of substitution for the inputs ϵ of the final good producers is set to match an average markup of 20%. I picked the price adjustment costs ξ to match the average price adjustment frequency of 12 months. Finally, the parameter of the Taylor rule ϕ_π is 1.2.

5.2 Model performance

In the previous section, I explained how I calibrate the model to match the data moments on residual wage dispersion and labor market flows. Now, I investigate the model's capacity to replicate stylized features of the business cycle under this parameter choice. To accomplish this, I explore the MIT shock and derive the model's solution in terms of the entire history of exogenous shocks, as proposed by Boppart, Krusell and Mitman (2018). After obtaining the solution, I simulate the model and calculate the business cycle properties of the main labor market flows in the simulated data. Results are presented in Table 2.

	y	v	f	u
Model				
Relative standard deviation	1.00	10.88	11.92	8.30
Correlation with output	1.00	0.91	0.95	-0.79
Correlation with unemployment	-0.79	-0.65	-0.71	1.00
Data				
Relative standard deviation	1.00	10.06	11.01	9.83
Correlation with output	1.00	0.59	0.86	-0.79
Correlation with unemployment	-0.79	-0.67	-0.93	1.00

Table 2: Labor market moments comparison between model and data

The model reasonably captures stylized features of the US labor market. It generates highly procyclical job-finding rates and vacancy postings, along with countercyclical fluctuations in unemployment, which align with the data. The model matches the volatility of job-finding rates and vacancy postings quite well but somewhat underperforms in terms of the volatility of unemployment. Overall, the model performs sufficiently well in replicating key aspects of the business cycle dynamics in the labor market.

Next, I am interested in the model's ability to replicate untargeted moments of labor earnings growth. As the income growth distribution provides valuable information about the underlying income process, matching the moments of labor income growth is crucial for establishing proper household incentives and generating a plausible level of precautionary saving.

For the empirical counterpart, I select the high-order moments of yearly labor earnings growth distribution reported by Guvenen et al. (2021). To ensure comparability, I construct the yearly labor income w^y_{it} as a sum of labor earnings over 12 months $w^y_{it} = \int_1^{12} w_{ih} dh$, with labor income growth defined as $\log w^y_{it+1} - \log w^y_{it}$. To obtain the income growth distribution, I simulate an income path of a worker for 100,000 periods in the aggregate stationary equilibrium. Table 3 reports the result.

The model effectively captures two key characteristics of income growth distribution: negative

Moment	Data	Model
$Var[\Delta w^y]$	0.26	0.11
$Skew[\Delta w^y]$	-1.07	-0.41
$Kurt[\Delta w^y]$	14.93	9.36
$Share \geq 2\sigma$	0.066	0.069
$Share \geq 3\sigma$	0.024	0.017

Table 3: Moments of income growth distribution

skewness and positive excess kurtosis. These characteristics arise from the interplay of two essential dynamics: the gradual advancements in labor earnings as workers reallocate into more productive matches, and the sudden setbacks caused by separation shocks, which effectively thrust individuals back to the lower rungs of the job ladder. Several works demonstrate that this disproportion between progressive increments and sudden reversals of labor earnings can generate negative skewness (Hubmer, 2018; Alves, 2022). Furthermore, this theoretical mechanism finds empirical support (Guvenen et al., 2021).

In the data, the labor earnings growth distribution demonstrates a larger skewness compared to what the model produces. Empirically, around half of the skewness in earnings originates from hours skewness, which the model abstracts from. Additionally, the model somewhat underperforms in terms of variance. This is because the model incorporates wage risk associated only with the job ladder, which accounts for only part of the labor income variance observed in the data. Notably, the model effectively replicates the high empirical density at the tails of the earnings growth distribution, resulting in high kurtosis.

5.3 Aggregate productivity shock

Next, I investigate the dynamic behavior of aggregates through a quantitative experiment. To this end, I perturb the economy with a zero-probability shock to aggregate productivity ("MIT" shock). Then, I allow productivity to converge deterministically toward its steady-state value.

Consider the economy remaining at a steady state before the shock occurs. At $t = 0$, all retailer firms experience an unexpected 1% decrease in productivity A_t. After the shock, productivity obeys the following law:

$$dA_t = \theta^A (A^{ss} - A_t) dt \tag{23}$$

where θ^A is a parameter of persistence. Figure 3 illustrates the economy's adjustment following the productivity shock in the baseline economy.

The initial adjustment of the economy is consistent with the mechanics of conventional HANK&SAM

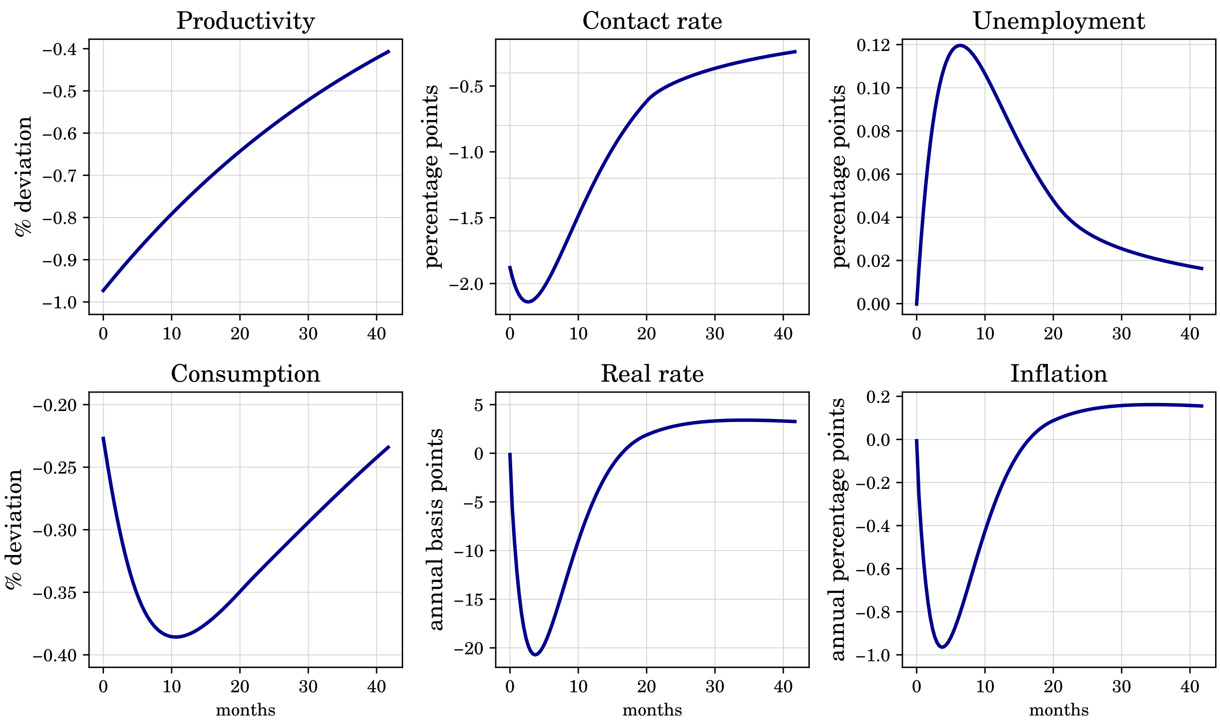


Figure 3: Response to a negative productivity shock in the baseline model

models extensively discussed in the literature. Following an adverse shock, a decline in productivity prompts retailers to decrease their demand for labor services, reducing the incentive for labor intermediaries to enter the market and post vacancies, which worsens aggregate labor market conditions. Low labor-market tightness has a dual effect on the dynamics of this economy, operating through (i) precautionary saving and (ii) compositional changes.

Initially, a weak labor market primarily impacts aggregate demand through the precautionary saving motive. As explained in Section 4.1, low contact rates increase both unemployment and wage risks, forcing risk-averse households to reduce their consumption. In a sticky-price economy, consumption cuts depress aggregate demand, prices, and interest rates, which in turn feed back into a weaker labor market. Thus, upon impact, the demand feedback stems mainly from the fear of unemployment and wage changes.

In contrast to models with zero liquidity, in economies with a non-degenerate wealth distribution, low labor market tightness influences aggregate demand through the composition effects as well. After one period, the economy's response is shaped not only by the income risk perceived by workers but also by actual compositional changes stemming from a weak labor market. As a result of this latter effect, approximately three quarters after the shock, the economy undergoes an overshoot before monotonically converging to its steady state.

5.4 Wage risk vs unemployment risk

To analyze the relative contributions of unemployment risk and wage risk in aggregate shock propagation, I construct two counterfactual economies: (i) one with wage risk held constant and (ii) another with overall labor income risk kept unchanged. Building on these counterfactuals, I further decompose the impact of wage risk by examining two additional scenarios: (a) a setting in which wages adjust proportionally, thus deactivating the relative wage channel, and (b) a case where the job-to-job transition rate remains fixed over time, effectively shutting down both the relative wage and reallocation channels.

Following the approach of Kaplan, Moll and Violante (2018), I begin by expressing aggregate consumption as a function of a series of labor market transition rates, wages, and interest rates entering the worker problem:

$$C_t\left(\left\{f_t^{ee}, w_t, f_t, r_t\right\}_{t\geq 0}\right) = \int c_t\left(a_t, \frac{w_{x,t}}{w_{n,t}}, w_{n,t}, \left\{f_t^{ee}, \frac{w_{x,t}}{w_{n,t}}, w_{n,t}, f_t, r_t\right\}_{t\geq 0}\right) d\Phi_t(a,x) \tag{24}$$

The key step is to rewrite individual consumption as a function of the wage in the most productive state ($w_{n,t}$) and the ratio of the wage in the current productivity state to the wage in the most productive state ($\frac{w_{x,t}}{w_{n,t}}$). Next, I calculate the total differential of Equation (24) with respect to all prices and transition rates that enter the worker problem, which allows me to decompose the consumption response at $t = 0$:

$$dC_0 = \int_0^\infty \frac{\partial C_t}{\partial f_t^{ee}} df_t^{ee} dt + \int_0^\infty \frac{\partial C_t}{\partial \frac{w_{x,t}}{w_{n,t}}} d\frac{w_{x,t}}{w_{n,t}} dt + \int_0^\infty \frac{\partial C_t}{\partial f_t} df_t dt + \int_0^\infty \frac{\partial C_t}{\partial w_{n,t}} dw_{n,t} dt + \int_0^\infty \frac{\partial C_t}{\partial r_t} dr_t dt \tag{25}$$

The first term on the right-hand side of Equation (25) is the partial consumption response to changes in the contact (job-to-job) rate f_t^{ee}, while the second term corresponds to the partial consumption response to changes in the relative wages $\frac{w_{x,t}}{w_{n,t}}$. Together, these two terms capture the dynamic impact of wage risk on aggregate consumption. The third term reflects the partial consumption response to changes in job-finding rates f_t, which encapsulates the unemployment risk in this economy. The remaining two terms capture the consumption responses to fluctuations in the overall wage level w_t and the interest rate r_t.

My main focus is on the first three terms of Equation (25). I isolate the contributions of wage and unemployment risks to the aggregate consumption response by constructing counterfactual equilibria in which one or more of these terms are held constant. I then compare these counterfactual equilibria to the baseline scenario, where all channels remain operative. I present the results in Figure 4.

In Figure 4, consider two counterfactual scenarios: one with wage risk held constant (represented by the green dash-dotted line) and another with both wage and unemployment risks held

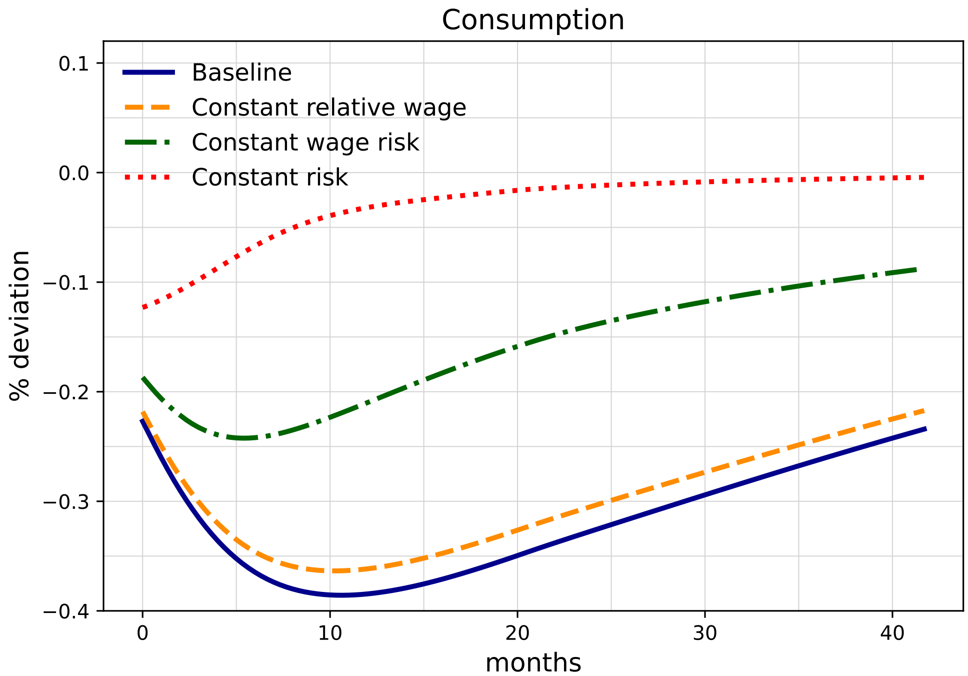


Figure 4: Impact of wage risk

constant (represented by the red dotted line), effectively eliminating overall idiosyncratic labor income risk in the latter case. Clearly, both unemployment and wage risks are significant drivers of the consumption response in this economy, together nearly doubling the on-impact aggregate consumption response. The relative contribution of wage risk is smaller, with approximately two-thirds of the overall income risk amplification attributed to unemployment risk. Nonetheless, the contribution of wage risk remains quantitatively important.

Next, I decompose the impact of wage risk into the contributions from the relative wage channel and the reallocation channel. When wages adjust proportionally (represented by the orange dashed line in Figure 4), the decline in consumption in response to productivity shocks is slightly less pronounced compared to the baseline scenario. Thus, the relative wage channel acts as a destabilizing force in the economy with positive liquidity, in contrast to the zero-liquidity case. This occurs because precautionary saving demand now arises not only from workers at the top of the job ladder but also from those in less productive matches and the unemployed. Nevertheless, I find that the majority of the amplification effect stems from the reallocation channel, as indicated by the difference between the orange and green lines. Note that the counterfactual path of the contact rate also affects consumption dynamics by altering the economy's composition, reducing the share of workers in high- and middle-productivity matches, and thereby distorting aggregate savings. I discuss the role of this composition effect in the next section.

Overall, the model highlights the significant quantitative impact of cyclical precautionary savings driven by wage risk. The relative wage and reallocation mechanisms together amplify the precautionary consumption response to productivity shocks by up to one-third compared to scenarios without cyclical wage risk.

5.5 Additional discussion

Compositional changes. In my economy, similar to a zero-liquidity case, *individual* consumption responses are determined by the relative strengths of precautionary saving and consumption-smoothing motives. However, assuming positive liquidity, the *aggregate* consumption response is further influenced by compositional changes in the economy, in addition to these two channels.

To understand the role of the composition effect in the aggregate demand response, in Appendix D, I contrast the aggregate saving functions in a HANK & SAM economy, with and without a job ladder. Given positive liquidity, labor market frictions affect aggregate savings not only by altering perceived uncertainty but also by causing compositional changes in the labor force. Following a negative productivity shock, the pool of unemployed workers expands, giving rise to what is known as the composition effect. While the exact magnitude of this effect typically depends on calibration, and especially on the tightness of borrowing constraints, under any realistic calibration, the composition effect acts as a stabilizing force in a conventional HANK & SAM economy. Cho (2023) discusses the quantitative implications of compositional changes for aggregate dynamics. In his estimated model, the composition effect dampens the precautionary demand feedback, leading to excessive consumption volatility, an issue that does not arise in zero-liquidity economies.

On the contrary, the introduction of the job ladder makes the sign of the composition channel ambiguous. Eased borrowing constraints for low-productivity workers can potentially reverse the standard composition effect if the mass of unemployed workers with little or no asset levels is substantial and new job matches are predominantly low-productivity. In this case, the composition effect generates a countercyclical supply of savings, acting as an additional destabilizing force. In Figure D.2 of Appendix D, I conduct another counterfactual experiment to isolate the dynamic changes in the compositional structure of the labor force in my model.

Frequency of renegotiation. In my model, wage contracts are not permanent and can be renegotiated infrequently. However, throughout the manuscript, I assume that the frequency of renegotiation approaches infinity, implying that wages are continuously renegotiated. This assumption allows the model to recover the standard result that, in a matched worker-firm pair, the total surplus is divided according to their respective bargaining powers.

If renegotiation becomes less frequent, the contracted wage remains fixed for a longer period of time. Renegotiation is an important determinant of a worker's value and, consequently, plays a key role in determining the contracted wage and the turnover rate. Gottfries (2018) demonstrates that when wages are renegotiated infrequently, workers, in equilibrium, tend to receive a larger share of the total surplus than their bargaining power would suggest. This occurs because firms benefit from the reduced turnover associated with longer wage contracts. In the Appendix, I show that the worker's share of the match surplus generally depends on the job-to-job transition rate and is therefore cyclical. However, these cyclical fluctuations are relatively small.

6 An Empirical Examination

The previous sections have explored both theoretically and quantitatively the role of wage risk arising from the cyclical job ladder in propagating aggregate shocks. Quantitatively, wage risk plays out through fluctuations in expected wage growth, driven by the probability of moving up the job ladder (the reallocation channel), and through variations in relative wages (the relative wage channel). In this section, I examine these channels empirically. Specifically, I compare the model-implied relationships between the job-to-job transition rate and wage growth with the patterns observed in the data. It is important to note that the goal here is not to formally validate the model, but rather to assess the empirical plausibility of both channels.

6.1 Theoretical link between wage growth and job-to-job movements

Before turning to the data, I first derive a theoretical relationship between wage growth and the job-to-job transition rate. To avoid unnecessary complications, I return to the labor market model introduced in Section 2, where the equilibrium wage does not depend on a worker's asset level. In general terms, the expected wage growth of an individual worker between time t and $t + dt$ can be expressed as the sum of the wage growth for job stayers and the wage growth for job switchers, weighted by their respective probabilities:

$$E(\dot{w}_t(x)) = f^{ee} \underbrace{\int_x^{\bar{x}} \left[w_t(s) - w_t(x) \right] dG(s)}_{\text{job-switchers}} + (1 - f^{ee}(1 - G(x)) \underbrace{\dot{w}_t(x)}_{\text{job-stayers}} \tag{26}$$

Note that the first term is necessarily nonnegative, as equilibrium in this economy can only exist for wages that are weakly increasing in idiosyncratic productivity, x. This brings me to the first key theoretical relationship: conditional on *job switching*, wage growth and job-to-job transitions should be *positively* correlated. This relationship is directly relevant to the reallocation channel: When the labor market is slack, lower job-job transition rates worsen job prospects and reduce expected income, which increases precautionary saving and further depresses economic activity.

Next, I examine the wage growth of job stayers. To derive the relationship between wage growth and the job-to-job transition rate for this subgroup of workers, I differentiate the wage equation (Equation 1) with respect to the job-to-job transition rate f^{ee}:

$$\frac{\partial w(x)}{\partial f^{ee}} = -\left[\phi \int_{x}^{\overline{x}} J(s)dG(s) + \phi f^{ee} \frac{\partial \int_{x}^{\overline{x}} J(s)dG(s)}{\partial f^{ee}}\right] \tag{27}$$

The first term in the brackets is positive, while the sign of the second term is generally ambiguous. As I demonstrate in Appendix B, for certain distributions, a closed-form expression for wages can be obtained. For example, in the case of uniform and other specific distributions, it can be shown that the sum of the two terms in brackets is positive, implying a *negative* correlation between wage growth and job-to-job transitions among *job stayers*. This relationship underpins the relative wage channel.

Before moving on to the empirical evaluation of these two theoretical relationships, two important remarks should be made. First, the negative link between wage growth and job-to-job transitions among job stayers diminishes as productivity increases. This is the essence of the relative wage channel: at the lower end of the job ladder, wages are sensitive to changes in the job-to-job transition rate, while at the upper end, they are less sensitive because the value of the integral decreases with x. This generates disproportional cyclical adjustments in wages across the productivity distribution, implying that the negative correlation should be observable at the bottom of the job ladder.

Second, this is a strong result that conflicts with some empirical findings. For example, studies by Karahan et al. (2017) and Moscarini and Postel-Vinay (2017*b*) document a positive correlation between wage growth and the job-to-job transition rate, highlighting the role of outside offers and wage renegotiations. Thus, the negative correlation in the data may be blurred by other channels.

6.2 Data and variable construction

I employ individual-level labor income data from the Survey of Income and Program Participation (SIPP), conducted by the U.S. Census Bureau. The SIPP, initiated in 1983, consists of monthly panel data that provide detailed information on labor market earnings and individual characteristics, with the advantage of allowing the identification of job-to-job transitions.

I concentrate on the SIPP surveys covering 1996-2013 where individual job-job transitions could be credibly identified. In defining employment status for a given month, I consider an individual to be employed if they report maintaining a job for the entire duration of the month. This definition includes those who experience paid or unpaid absences for reasons such as vacations, illnesses, or involvement in labor disputes. However, it specifically excludes individuals who report being laid off, those without any work, or those who have been out of the labor force for at least one full week during the month. Similarly, I define an individual as unemployed if they report being without work and actively searching for a job throughout the entire month.

Since there is no standard definition of the job-to-job transition rate in the literature, I adopt the approach of Tjaden and Wellschmied (2014), who identifies job-to-job movements as either transitions between firms or changes in occupation. To identify transitions between firms, I use monthly employer identifiers, while occupational changes are tracked using a three-digit occupational code.

I construct a wage growth variable for each individual i at time t, calculated as the month-to-month log difference in wages. Additionally, I define two job movement indicators: UE_{it} (unemployment to employment) and EE_{it} (employment to employment), which are set to one if the corresponding transition occurs between t and $t+1$, and zero otherwise.

6.3 Methodology

Next, I estimate the empirical relationship between wage growth and job-to-job transition rates and compare it with the theoretical predictions of my model. To achieve this, I follow the methodology outlined by Moscarini and Postel-Vinay (2017*b*), applying a two-step procedure. The core idea is to assess the correlation between wage growth and the labor market flow fixed effects for each market (second step), after first filtering observations to control for composition effects (first step).

Step 1. *Elimination of the composition effect*. My three variables of interest are wage growth (Δw_{it}), unemployment-employment (UE_{it}), and employment-employment (EE_{it}) transition. For each individual i, y_{it} denotes either the wage growth or the labor market transition indicator. I estimate the following equation:

$$y_{it} = X_{it}^{y}\beta_y + M_{it}^{t}\gamma_y + \varepsilon_{it} \tag{28}$$

where M is the interaction of dummies for age, race, gender, education, and time; X is a vector of individual characteristics of state of residence, occupation, industry, employer size, and union status. Then, I extract the market-time fixed effects:

$$\hat{\Phi}_{it}^{y} = M_{it}^{t}\hat{\gamma}_y \tag{29}$$

Step 2. *Estimating the correlation between the fixed effects*. Next, I regress the time-fixed effect of the wage growth on the time-fixed effects of labor market flows

$$\hat{\Phi}_{it}^{\Delta w} = \sum_{h\in\{UE,EE\}} \hat{\Phi}_{it}^{h}\delta_h + M_{it}\psi + \phi t + \eta_{it} \tag{30}$$

where t is the time dummy and η_{it} stands for the error term.

6.4 Results and discussion

Table 4: Regression Results: Log Change in Monthly Nominal Earnings

Dependent Variable:	Log Change in Monthly Nominal Earnings			
	(1) All	(2) Stayers	(3) Stayers	(4) Stayers
EE Rate	0.0424	0.00806	-0.01555	0.00158
	(0.0006)	(0.00026)	(0.00043)	(0.00091)
UE Rate	-0.0005	-0.00136	-0.00132	-0.00139
	(0.00004)	(0.00002)	(0.00002)	(0.00002)
Interaction Terms: EE Rate				
× Age 36-45			0.07746	
			(0.00062)	
× Age 46-60			0.00309	
			(0.00069)	
× Age 60+			-0.02368	
			(0.00100)	
× Educ2				-0.02205
				(0.00128)
× Educ3				-0.01516
				(0.00121)
× Educ4				-0.00559
				(0.00112)
× Educ5				0.03979
				(0.00097)
Observations	all	stayers	stayers	stayers

The table presents the result of estimating Equation (30). The first column presents results for the whole sample, columns 2-4 are for job stayers. Data is from the Survey of Income and Program Participation covering 1996-2013 at a monthly frequency.

The results of the final step estimation are presented in Table 4. The first column shows the relationship between changes in individual wage growth and changes in both the job-finding and job-to-job transition rates for all workers in the sample. The positive and statistically significant coefficient for the job-to-job transition (EE) rate indicates a strong positive correlation between job-to-job transitions and wage growth, consistent with the theoretical hypothesis from my model. When the job-to-job transition rate is high, workers ascend the job ladder more quickly and undergo more substantial wage increases.

Importantly, the correlation between the job-to-job transition rate and wage growth surpasses that of the job-finding rate. This result suggests that wage growth is driven more by the cyclical dynamics of the job ladder than by fluctuations in the value of outside options, as described in the framework proposed by Mortensen and Pissarides (1994). These findings are consistent with those of Faberman, Justiniano et al. (2015), Moscarini and Postel-Vinay (2017*b*), and Karahan et al. (2017), who demonstrated that the job-to-job transition rate is a far better predictor of wage growth than the job-finding rate. Moreover, this provides empirical support for the strand of labor market theories originating from Burdett and Mortensen (1998).

The second column of Table 4 presents the estimates for the same specification, but excluding job switchers. Similarly, I find a positive correlation between wage growth and job-to-job transition rates within the subsample of job stayers. This result aligns with the perspective of Postel-Vinay and Robin (2002), which suggests that wage growth among job stayers may be influenced by outside offers, leading to wage renegotiations.

In columns 3 and 4, I analyze the same specification, this time controlling for age and education groups. The correlation between wage growth and the job-to-job transition rate is found to be negative for both the youngest and oldest age groups, as well as for workers with lower levels of education. I interpret this negative correlation as empirical evidence supporting my relative wage channel. Specifically, changes in the likelihood of poaching, while controlling for other labor flows, result in wage reductions for workers in low-productivity matches. This leads to a negative correlation between wage growth and the job-to-job transition rate, particularly for workers at the lower end of the job ladder. However, this relationship could be obscured by the renegotiation channel, so the negative correlation might persist only among certain groups of workers.

As a robustness check, I test several alternative specifications using different definitions of wage growth, controlling for other labor flows, and analyzing various subsamples. Appendix E shows that the main result remains robust across these modifications.

7 Conclusion

This paper explores the role of cyclical wage risk in transmitting aggregate shocks, adding a different perspective to the existing literature that predominantly focuses on unemployment risk. In my model, wage risk arises from cyclical reallocations across the job ladder. Within a zero-liquidity economy, I theoretically identify two channels through which the job ladder propagates the aggregate shocks: the reallocation and relative wage channels, which I argue are empirically plausible. Then, I demonstrate that both channels significantly impact cyclical dynamics even in more realistic setups. The quantitative model, which relaxes the assumption of zero liquidity, illustrates that the cyclical nature of wage risk enhances demand feedback, thereby amplifying employment and consumption responses by roughly one-third. This finding suggests that other aspects of the frictional labor market, beyond conventional unemployment risk, can also play an important role in propagating aggregate shocks when combined with imperfections in goods and financial markets.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. 2022. "Income and wealth distribution in macroeconomics: A continuous-time approach." *The review of economic studies*, 89(1): 45–86.

Alves, Felipe. 2022. "Job ladder and business cycles." Bank of Canada.

Autor, David H, Lawrence F Katz, and Melissa S Kearney. 2008. "Trends in US wage inequality: Revising the revisionists." *The Review of economics and statistics*, 90(2): 300–323.

Bayer, Christian, Ralph Lütticke, Lien Pham-Dao, and Volker Tjaden. 2019. "Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk." *Econometrica*, 87(1): 255–290.

Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky. 1986. "The Nash bargaining solution in economic modelling." *The RAND Journal of Economics*, 176–188.

Birinci, Serdar, Fatih Karahan, Yusuf Mercan, and Kurt See. 2022. "Labor Market Shocks and Monetary Policy." *FRB St. Louis Working Paper*, , (2022-16).

Bonciani, Dario, and Joonseok Oh. 2021. "Unemployment risk, liquidity traps, and monetary policy." *Liquidity Traps, and Monetary Policy (February 27, 2021)*.

Boppart, Timo, Per Krusell, and Kurt Mitman. 2018. "Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative." *Journal of Economic Dynamics and Control*, 89: 68–92.

Broer, Tobias, Jeppe Druedahl, Karl Harmenberg, and Erik Öberg. 2021. "The unemployment-risk channel in business-cycle fluctuations."

Burdett, Kenneth, and Dale T Mortensen. 1998. "Wage differentials, employer size, and unemployment." *International Economic Review*, 257–273.

Challe, Edouard. 2020. "Uninsured unemployment risk and optimal monetary policy in a zero-liquidity economy." *American Economic Journal: Macroeconomics*, 12(2): 241–83.

Challe, Edouard, and Xavier Ragot. 2016. "Precautionary saving over the business cycle." *The Economic Journal*, 126(590): 135–164.

Challe, Edouard, Julien Matheron, Xavier Ragot, and Juan F Rubio-Ramirez. 2017. "Precautionary saving and aggregate demand." *Quantitative Economics*, 8(2): 435–478.

Cho, Daeha. 2023. "Unemployment risk, MPC heterogeneity, and business cycles." *Quantitative Economics*, 14(2): 717–751.

Faberman, R Jason, Alejandro Justiniano, et al. 2015. "Job switching and wage growth." *Chicago Fed Letter*.

Faccini, Renato, and Leonardo Melosi. 2021. "Bad jobs and low inflation."

Fujita, Shigeru, Giuseppe Moscarini, and Fabien Postel-Vinay. 2020. "Measuring employer-to-employer reallocation." National Bureau of Economic Research.

Gottfries, Axel. 2018. "Bargaining with renegotiation in models with on-the-job search." *Available at SSRN 3058870*.

Graves, Sebastian. 2020. "Does Unemployment Risk Affect Business Cycle Dynamics?" *International Finance Discussion Paper*, , (1298).

Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song. 2021. "What do data on millions of US workers reveal about lifecycle earnings dynamics?" *Econometrica*, 89(5): 2303–2339.

Hubmer, Joachim. 2018. "The job ladder and its implications for earnings risk." *Review of Economic Dynamics*, 29: 172–194.

Jung, Euiyoung. 2023. "Wage rigidity and destabilizing spirals." *Journal of Macroeconomics*, 77: 103546.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante. 2018. "Monetary policy according to HANK." *American Economic Review*, 108(3): 697–743.

Karahan, Fatih, Ryan Michaels, Benjamin Pugsley, Ayşegül Şahin, and Rachel Schuh. 2017. "Do job-to-job transitions drive wage fluctuations over the business cycle?" *American Economic Review*, 107(5): 353–357.

Krusell, Per, Toshihiko Mukoyama, and Ayşegül Şahin. 2010. "Labour-market matching with precautionary savings and aggregate fluctuations." *The Review of Economic Studies*, 77(4): 1477–1507.

Mortensen, Dale. 2003. *Wage dispersion: why are similar workers paid differently?* MIT press.

Mortensen, Dale T, and Christopher A Pissarides. 1994. "Job creation and job destruction in the theory of unemployment." *The review of economic studies*, 61(3): 397–415.

Moscarini, Giuseppe, and Fabien Postel-Vinay. 2017*a*. "The job ladder: Inflation vs. reallocation."

Moscarini, Giuseppe, and Fabien Postel-Vinay. 2017*b*. "The relative power of employment-to-employment reallocation and unemployment exits in predicting wage growth." *American Economic Review*, 107(5): 364–368.

Pissarides, Christopher A. 1994. "Search unemployment with on-the-job search." *The Review of Economic Studies*, 61(3): 457–475.

Postel-Vinay, Fabien, and Jean-Marc Robin. 2002. "Equilibrium wage dispersion with worker and employer heterogeneity." *Econometrica*, 70(6): 2295–2350.

Ravn, Morten O, and Vincent Sterk. 2017. "Job uncertainty and deep recessions." *Journal of Monetary Economics*, 90: 125–141.

Ravn, Morten O, and Vincent Sterk. 2021. "Macroeconomic fluctuations with HANK & SAM: An analytical approach." *Journal of the European Economic Association*, 19(2): 1162–1202.

Shimer, Robert. 2005. "The cyclical behavior of equilibrium unemployment and vacancies." *American economic review*, 95(1): 25–49.

Shimer, Robert. 2006. "On-the-job search and strategic bargaining." *European Economic Review*, 50(4): 811–830.

Tjaden, Volker, and Felix Wellschmied. 2014. "Quantifying the contribution of search to wage inequality." *American Economic Journal: Macroeconomics*, 6(1): 134–161.

Werning, Iván. 2015. "Incomplete markets and aggregate demand." National Bureau of Economic Research.

Precautionary saving, wage risk, and cyclical reallocation
Appendix

Mykhailo Matvieiev

A Analytical model

This appendix presents the derivations of the results from the analytical section. Specifically, I derive a wage equation for the model with endogenous destruction, which includes the model with exogenous destruction as a special case.

Each worker-firm match has an aggregate productivity z and an idiosyncratic productivity x. With Poisson intensity λ, an idiosyncratic shock (without memory) hits a match and changes x to some random value from distribution $dG(x)$. Firms have a reservation productivity R and destroy a job if $x < R$. From this setup, it is straightforward to move to a model with exogenous destruction by assuming that every idiosyncratic shock leads to the destruction of a match. Both models deliver the same wage equations.

Denoting U as the value of being unemployed and W as the value of being employed, the Hamiltonian-Jacobi-Bellman equations for the unemployed and employed workers are:

$$(r+\lambda)W(x) = w(x) + \lambda \int_R^{\bar{x}} W(s)dG(s) + \lambda G(R)U + \theta q(\theta)\left[\int_x^{\bar{x}} W(s)dG(s) - (1-G(x))W(x)\right]$$

$$rU = b + \theta q(\theta) \int_R^{\bar{x}} \left[W(s) - U\right] dG(s)$$

Denoting J and V as the values of firms with filled and vacant jobs, respectively, the Hamiltonian-Jacobi-Bellman (HJB) equations for these firms are:

$$rJ(x) = zx - w(x) + \lambda \int_R^{\bar{x}} J(s)dG(s) - ((1-G(x))\theta q(\theta) + \lambda)J(x)$$

$$rV = -zc + q(\theta)\left[\int_R^{\bar{x}} F(s)[J(s) - V]dG(s)\right]$$

Wage determined according to rule $\frac{W(x)-U}{J(x)} = \frac{\phi}{1-\phi}$, where ϕ is bargaining power or equivalently: $(1-\phi)W(x) - \phi J(x) = (1-\phi)U$. Consequently,

$$(r+\lambda+\theta q(\theta)(1-G(x)))(1-\phi)U =$$
$$(1-\phi)\left[w(x) + \lambda \int_R^{\bar{x}} W(s)dG(s) + \lambda G(R)U + \theta q(\theta)\int_x^{\bar{x}} W(s)dG(s)\right] - \phi\left[zx - w(x) + \lambda \int_R^{\bar{x}} J(s)dG(s)\right] =$$
$$w(x) - \phi zx + (1-\phi)\lambda G(R)U + \lambda\left[(1-\phi)\int_R^{\bar{x}} W(s)dG(s) - \phi \int_R^{\bar{x}} J(s)dG(s)\right] + (1-\phi)\theta q(\theta)\int_x^{\bar{x}} W(s)dG(s) =$$

$$w(x) - \phi z x + (1-\phi)\lambda G(R) U + \lambda \left[(1-\phi) \int_R^{\bar{x}} U dG(s) \right] + (1-\phi)\theta q(\theta) \int_x^{\bar{x}} W(s) dG(s) =$$

$$w(x) - \phi z x + (1-\phi)\lambda G(R) U + \lambda (1-\phi) U [G(\bar{x}) - G(R)] + (1-\phi)\theta q(\theta) \int_x^{\bar{x}} W(s) dG(s) =$$

$$w(x) - \phi z x + \lambda(1-\phi) U + (1-\phi)\theta q(\theta) \int_x^{\bar{x}} W(s) dG(s)$$

$$(r + \lambda + \theta q(\theta)(1 - G(x)))(1-\phi) U = w(x) - \phi z x + \lambda(1-\phi) U + (1-\phi)\theta q(\theta) \int_x^{\bar{x}} W(s) dG(s)$$

$$(r + \theta q(\theta)(1 - G(x)))(1-\phi) U = w(x) - \phi z x + (1-\phi)\theta q(\theta) \int_x^{\bar{x}} W(s) dG(s)$$

$$r(1-\phi) U = w(x) - \phi z x + (1-\phi)\theta q(\theta) \int_x^{\bar{x}} [W(s) - U] dG(s)$$

Now using the value for the unemployed:

$$(1-\phi)\left[b + \theta q(\theta) \int_R^{\bar{x}} \left[W(s) - U \right] dG(s) \right] = w(x) - \phi z x + (1-\phi)\theta q(\theta) \int_x^{\bar{x}} [W(s) - U] dG(s)$$

$$(1-\phi) b = w(x) - \phi z x - (1-\phi)\theta q(\theta) \int_R^{x} [W(s) - U] dG(s)$$

$$w(x) = (1-\phi) b + \phi z x + (1-\phi)\theta q(\theta) \int_R^{x} [W(s) - U] dG(s)$$

$$w(x) = (1-\phi) b + \phi z x + \phi \theta q(\theta) \int_R^{x} J(s) dG(s)$$

$$w(x) = (1-\phi) b + \phi z x + \phi \theta q(\theta) \int_R^{\bar{x}} J(s) dG(s) - \phi \theta q(\theta) \int_x^{\bar{x}} J(s) dG(s)$$

now using the FEC obtain:

$$w(x) = (1-\phi) b + \phi z (x + \theta c F(\xi)^{-1}) - \underbrace{\phi \theta q(\theta) \int_x^{\bar{x}} J(s) dG(s)}_{\Delta}$$

Next, the goal is to find Δ. Taking HJB of employed workers and dividing both parts by $\theta q(\theta)$:

$$\frac{r + \lambda + (1 - G(x))\theta q(\theta)}{\theta q(\theta)} W(x) = \frac{1}{\theta q(\theta)} \left[w(x) + \lambda \int_R^{\bar{x}} \left[W(s) - U \right] dG(s) + \lambda U \right] + \int_x^{\bar{x}} W(s) dG(s)$$

Denoting:
$C(x) = \frac{r+\lambda+(1-G(x))\theta q(\theta)}{\theta q(\theta)}$, which depends on x through $G(x)$
$A(x) = \frac{1}{\theta q(\theta)} \left[w(x) + \lambda \int_R^{\bar{x}} \left[W(s) - U \right] dG(s) + \lambda U \right]$, which depends on x through $w^s(x)$
Rewrite:

$$C(x) W(x) = A(x) + \int_x^{\bar{x}} W(s) dG(s)$$

$$C'(x) W(x) + C(x) W'(x) = A(x) - W(x) G'(x)$$

$$-G'(x) W(x) + C(x) W'(x) = A'(x) - W(x) G'(x)$$

$$C(x) W'(x) = \frac{w'(x)}{\theta q(\theta)}$$

Maximization of Nash product implies:

$$W'(x) = \frac{\phi}{(1-\phi)} J'(x)$$

From the previous wage equation

$$w'(x) = \phi p + \phi\theta q(\theta) J(x) G'(x)$$

Plug in $W'(x)$ and $w'(x)$

$$\begin{aligned}
C(x)\frac{\phi}{(1-\phi)} J'(x) &= \frac{\phi}{\theta q(\theta)} p + \phi J(x) G'(x) \\
C(x)\frac{\phi}{(1-\phi)} J'(x) &= \frac{\phi}{\theta q(\theta)} p - \phi J(x) C'(x) \\
C(x) J'(x) + (1-\phi) J(x) C'(x) &= \frac{(1-\phi)}{\theta q(\theta)} p
\end{aligned}$$

The RHS does not depend on x. First, I solve for the case where RHS = 0

$$\frac{J'(x)}{J(x)} = -(1-\phi)\frac{C'(x)}{C(x)}$$

$$J(x) = \Gamma C(x)^{-(1-\phi)}$$

Using the method of variation

$$J'(x) = \Gamma' C(x)^{-(1-\phi)} - (1-\phi)\Gamma C(x)^{-(1-\phi-1)} C'(x)$$

$$C(x) J'(x) = \Gamma' C(x)^{-(1-\phi)} C(x) - (1-\phi)\Gamma C(x)^{-(1-\phi)} C'(x)$$

Plug in the equation

$$\begin{aligned}
&\Gamma' C(x)^{-(1-\phi)} C(x) - (1-\phi)\Gamma C(x)^{-(1-\phi)} C'(x) + (1-\phi) J(x) C'(x) = \frac{(1-\phi)}{\theta q(\theta)} z \\
&\Gamma' C(x)^{-(1-\phi)} C(x) - (1-\phi)\Gamma C(x)^{-(1-\phi)} C'(x) + (1-\phi) J(x) C'(x) = \frac{(1-\phi)}{\theta q(\theta)} z \\
&\Gamma' C(x)^{\phi} = \frac{(1-\phi)}{\theta q(\theta)} z \\
&\Gamma = \frac{(1-\phi)}{\theta q(\theta)} z \int_R^x \frac{1}{C(x)^{\phi}} dx
\end{aligned}$$

To repeat, $\Delta = \phi\theta q(\theta)\int_x^{\bar{x}} J dG(s)$

To solve for Δ, an exact functional form for G is needed. Assuming uniform distribution U[0,$\bar{x}$].

$$\begin{aligned}
C(x) &= \frac{r+\lambda}{\theta q(\theta)} + 1 - G(x) = \frac{r+\lambda}{\theta q(\theta)} + 1 - \frac{x}{\bar{x}} \\
dC(x) &= -\frac{1}{\bar{x}} dx
\end{aligned}$$

$$\int_R^x \frac{1}{C^\phi}dx = \bar{x}\int_R^x \frac{1}{C^\phi}dC = \bar{x}\frac{1}{\phi-1}\frac{1}{C^{\phi-1}}\Bigg|_{C(x)}^{C(R)} = \bar{x}\frac{1}{\phi-1}\big(-C(x)^{1-\phi}+C(R)^{1-\phi}\big)$$

$$J = \frac{\bar{x}p}{\theta q(\theta)}\left[-1+\left[\frac{C(R)}{C(x)}\right]^{1-\phi}\right]$$

$$\Delta = \phi\bar{x}p\int_{\bar{x}}^x\left[-1+\left[\frac{C(R)}{C(x)}\right]^{1-\phi}\right]dG$$

$$\frac{\Delta}{\phi\bar{x}z} = \int_{\bar{x}}^x\left[1-\left[\frac{C(R)}{C(x)}\right]^{1-\phi}\right]dC = 1-\frac{x}{\bar{x}}-C(R)^{1-\phi}\int_{\bar{x}}^x C(x)^{-(1-\phi)}dC =$$
$$= 1-\frac{x}{\bar{x}}-C(R)^{1-\phi}\frac{1}{\phi}\Big(C(\bar{x})^\phi - C(x)^\phi\Big)$$

$$\Delta = \phi z(\bar{x}-x)+\bar{x}zC(R)\left[\frac{C(x)^\phi - C(\bar{x})^\phi}{C(\bar{R})^\phi}\right]$$

B Wage Equation

First I repeat HJB equations for employed workers and firms with filled jobs

$$\rho W_{a,x,t} = \max_{\{c\}}\left\{u(c)+\partial_a W_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c\big]+f_t^{ee}\int_x^{\overline{x}}g(x')\big[W_{a,x',t}-W_{a,x,t}\big]dx'+\lambda\big[W_{a,0,t}-W_{a,x,t}\big]+\partial_t W_{a,x,t}\right\}$$
$$r_t J_{a,x,t} = p_t^I x - w_{a,x,t}+\partial_a J_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c\big]+f_t^{ee}\int_x^{\overline{x}}g(x')\big[V_t-J_{a,x,t}\big]dx'+\lambda\big[V_t-J_{a,x,t}\big]+\partial_t J_{a,x,t}$$

Denoting optimal consumption as c^* and collecting terms:

$$\underbrace{\big[\rho+\lambda+f_t^{ee}\int_x^{\overline{x}}g(x')dx'\big]}_{M_{x,t}^C}W_{a,x,t} = u(c^*)+\partial_a W_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c^*\big]+f_t^{ee}\int_x^{\overline{x}}g(x')W_{a,x',t}dx'+\lambda W_{a,0,t}+\partial_t W_{a,x,t}$$

$$\underbrace{\big[r_t+\lambda+f_t^{ee}\int_x^{\overline{x}}g(x')dx'\big]}_{M_{x,t}^F}J_{a,x,t} = p_t^I x - w_{a,x,t}+\partial_a J_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c^*\big]+\partial_t J_{a,x,t}$$

$$W_{a,x,t} = \frac{u(c^*)+\partial_a W_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c^*\big]+f_t^{ee}\int_x^{\overline{x}}g(x')W_{a,x',t}dx'+\lambda W_{a,0,t}+\partial_t W_{a,x,t}}{M_{x,t}^C}$$
$$J_{a,x,t} = \frac{p_t^I x - w_{a,x,t}+\partial_a J_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c^*\big]+\partial_t J_{a,x,t}}{M_{x,t}^F}$$

Surplus-sharing rule $(1-\phi)\big[W_{a,x,t}-W_{a,0,t}\big] = \phi\big[J_{a,x,t}-V_t\big]$ for $x_t\in[\underline{x},\overline{x}]$ implies that

$$(1-\phi)\frac{u(c^*)+\partial_a W_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c^*\big]+f_t^{ee}\int_x^{\overline{x}}g(x')W_{a,x',t}dx'-(M_{x,t}^C-\lambda)W_{a,0,t}+\partial_t W_{a,x,t}}{M_{x,t}^C} =$$
$$\phi\frac{p_t^I x - w_{a,x,t}+\partial_a J_{a,x,t}\big[r_t\cdot a+w_{a,x,t}-c^*\big]+\partial_t J_{a,x,t}}{M_{x,t}^F}$$

Expressing $w_{a,x,t}$:

$$(1-\phi)\frac{\partial_a W_{a,x,t}}{M^C_{x,t}}w_{a,x,t} + (1-\phi)\frac{u(c^*) + \partial_a W_{a,x,t}\left[r_t \cdot a - c^*\right] + f^{ee}_t \int_x^{\overline{x}} g(x')W_{a,x',t}dx' - (M^C_{x,t} - \lambda)W_{a,0,t} + \partial_t W_{a,x,t}}{M^C_{x,t}} =$$

$$\phi\frac{-1 + \partial_a J_{a,x,t}}{M^F_{x,t}}w_{a,x,t} + \phi\frac{p^I_t x + \partial_a J_{a,x,t}\left[r_t \cdot a - c^*\right] + \partial_t J_{a,x,t}}{M^F_{x,t}}$$

$$\Bigg[\underbrace{(1-\phi)\frac{\partial_a W_{a,x,t}}{M^C_{x,t}} - \phi\frac{-1 + \partial_a J_{a,x,t}}{M^F_{x,t}}}_{M^W_{a,x,t}}\Bigg]w_{a,x,t} = \phi\frac{p^I_t x + \partial_a J_{a,x,t}\left[r_t \cdot a - c^*\right] + \partial_t J_{a,x,t}}{M^F_{x,t}}$$

$$-(1-\phi)\frac{u(c^*) + \partial_a W_{a,x,t}\left[r_t \cdot a - c^*\right] + f^{ee}_t \int_x^{\overline{x}} g(x')W_{a,x',t}dx' - (M^C_{x,t} - \lambda)W_{a,0,t} + \partial_t W_{a,x,t}}{M^C_{x,t}}$$

$$w_{a,x,t} = \phi\frac{p^I_t x + \partial_a J_{a,x,t}\left[r_t \cdot a - c^*\right] + \partial_t J_{a,x,t}}{M^W_{a,x,t}M^F_{x,t}}$$

$$-(1-\phi)\frac{u(c^*) + \partial_a W_{a,x,t}\left[r_t \cdot a - c^*\right] + f^{ee}_t \int_x^{\overline{x}} g(x')W_{a,x',t}dx' - (M^C_{x,t} - \lambda)W_{a,0,t} + \partial_t W_{a,x,t}}{M^W_{a,x,t}M^C_{x,t}} =$$

$$\phi\frac{p^I_t x + \partial_a J_{a,x,t}\left[r_t \cdot a - c^*\right] + \partial_t J_{a,x,t}}{M^W_{a,x,t}M^F_{x,t}} - (1-\phi)\frac{u(c^*) + \partial_a W_{a,x,t}\left[r_t \cdot a - c^*\right] + f^{ee}_t \int_x^{\overline{x}} g(x')W_{a,x',t}dx' - r_t W_{a,0,t} + \partial_t W_{a,x,t}}{M^W_{a,x,t}M^C_{x,t}} -$$

$$-\frac{1}{M^W_{a,x,t}M^C_{x,t}}\int_x^{\overline{x}} \phi f^{ee}_t J_{a,x',t}dG(x')$$

C Zero-liquidity economy

In the limiting case of a zero-liquidity economy, let's consider a scenario where events are timed differently within a period. Initially, at the start of the period, workers encounter the risk of separation with the probability λ. The remaining period is divided into two subperiods. During each subperiod, unemployed workers have a chance of finding a job in either a low-productivity match with probability $f^l_t \frac{1}{2}$, or a high-productivity match with probability $f^h_t \frac{1}{2}$. For workers in low-productivity matches, there exists a probability, $f^{ee}\frac{dt}{2}$, of transitioning to a more productive job within a subperiod.

In this timing convention, for a worker in the high-productivity state at the beginning of time t, the probability of transitioning to the unemployment state at the beginning of time $t+1$ is $\lambda(1 - f^h_t - f^l_t)$. The corresponding probability of transitioning to the low-productivity state at the beginning of time $t+1$ is $\lambda\frac{f^l_t}{2}((1 - \frac{f^{ee}_t}{2}) + (1 - \frac{f_t}{2}))$. Defining $f_t = f^h_t + f^l_t$ yields the probabilities from Equation (21).

D The composition effect

Standard HANK & SAM economy. Denoting saving policy by $\tilde{s}(.)$, the aggregate net savings supply in a standard HANK & SAM economy can be expressed as a sum of savings of employed and unemployed workers: $\int \tilde{s}(a,e)da + \int \tilde{s}(a,u)da$. With the unemployment rate denoted as u, we can rewrite the aggregate savings as $(1-u)\frac{\int \tilde{s}(a,e)da}{1-u} + u\frac{\int \tilde{s}(a,u)da}{u}$, and ultimately as $(1-u)\bar{s}^e + u\bar{s}^u$, where $\bar{s}^e$ and $\bar{s}^e$ represent the mean savings of employed and unemployed workers, respectively.

Next, let's consider a shock leading to an increase in the unemployment rate, such as a negative productivity shock. We can derive a change in aggregate savings S^a in response to this shock:

$$\frac{\partial S^a}{\partial u} = \underbrace{(1-u)\frac{\partial \bar{s}^e}{\partial u}}_{\text{change in savings employed}} + \underbrace{u\frac{\partial \bar{s}^u}{\partial u}}_{\text{change in savings unemployed}} + \underbrace{\bar{s}^u - \bar{s}^e}_{\text{composition effect}} \tag{D.1}$$

The first term in Equation (D.1) encapsulates the adjustment in savings of employed workers in response to the shock. The direction of this adjustment hinges on whether the consumption smoothing or precautionary channel prevails, akin to the zero-liquidity case discussed in Section 4.1. The second term represents the savings response of the unemployed, governed by aversion to intertemporal substitution and aversion to risk, much like the adjustment observed in employed workers. This term appears in the economy with positive liquidity because unemployed workers are not necessarily borrowing-constrained.

The last two terms in Equation (D.1) account for the disparity in savings between unemployed and employed workers. This disparity arises because search and matching frictions affect the economy not only by changing perceived uncertainty but also by inducing compositional changes in the labor force. The negative productivity shock leads to an increase in the pool of unemployed, giving rise to the composition effect. In any realistic calibration, $\bar{s}^u \leq \bar{s}^e$, but

In this context, a zero liquidity economy represents a limiting case where $\bar{s}^u = \bar{s}^e$ and compositional changes affect the economy solely through the dynamic path of labor market tightness.

HANK & SAM economy with OJS. The economy with a job ladder introduces an additional term in the composition effect. The change in aggregate savings now becomes:

$$\frac{\partial S^a}{\partial u} = \underbrace{\gamma^h(1-u)\frac{\partial \bar{s}^h}{\partial u}}_{\text{change in savings high-employed}} + \underbrace{\gamma^l(1-u)\frac{\partial \bar{s}^l}{\partial u}}_{\text{change in savings low-employed}} + \underbrace{u\frac{\partial \bar{s}^u}{\partial u}}_{\text{change in savings unemployed}} + \underbrace{\bar{s}^u - \gamma^h \bar{s}^h - \gamma^l \bar{s}^l}_{\text{composition effect}} \tag{D.2}$$

γ_l and γ_h represent the share of workers in low- and high-productivity states within the pool of the employed, while $\bar{s}^l$ and $\bar{s}^h$ denote the respective mean saving policies.

Returning to my quantitative mode, examine the steady-state saving function across the three employment states: the saving function for unemployed households, the saving function for employed workers at the bottom of the job ladder (low-productivity matches), and the saving function for employed workers at the top of the job ladder (high-productivity matches). I depict all three saving functions in Figure D.1.

While the savings of the employed in a high-productivity state always exceed the savings of the unemployed, expressed as $\bar{s}^u \leq \bar{s}^h$, this isn't necessarily the case for the low-productivity state. We observe that in the vicinity of the unemployed's borrowing limit, $\bar{s}^u > \bar{s}^l$. The reason behind this lies in the fact that the tightness of the borrowing constraint depends on the employment state of a worker, reflecting the ease of obtaining borrowing when a worker is more productive.

The sign of the composition channel becomes ambiguous in this environment. Eased borrowing constraints for low-productivity workers can potentially reverse the standard impact of compositional changes observed in a conventional HANK&SAM model without the on-the-job search. Intuitively, if the mass of unemployed workers with lower asset levels is significant, and new job matches predominantly have low

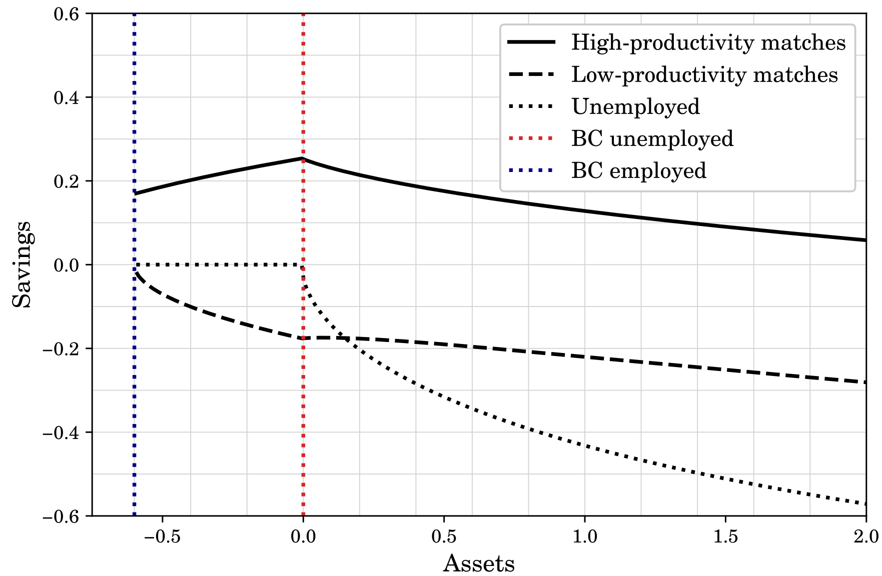


Figure D.1: Policy function

productivity, the composition effect generates a countercyclical supply of savings. This, in turn, acts as a destabilizing force.

I conduct another counterfactual experiment to disentangle the dynamic effect of compositional changes in my model. To this end, I simulate the transition path of the economy following the shock by assuming a fixed distribution of workers across income and wealth. The results of this experiment are illustrated in Figure D.2.

In a scenario with a fixed distribution, the economy converges to its steady state monotonically, contrasting with the baseline case. In the model where the composition effect is operative, the transition of workers from low-productivity jobs to unemployment generates an additional supply of savings. Over time, as the share of unemployed workers begins to converge back to a steady state, the change in composition triggers opposite dynamics, leading to an overshoot in the interest rate and inflation.

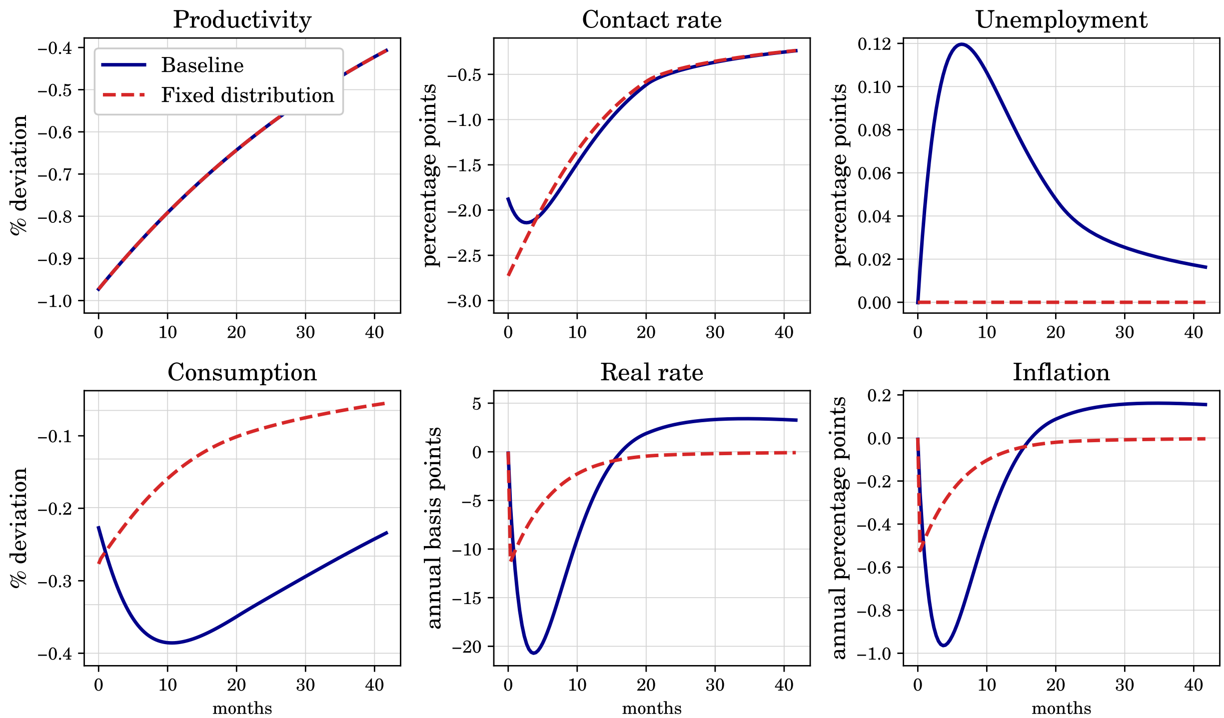


Figure D.2: Effect of compositional changes