Supply Shocks in the Fog: The Role of Endogenous Uncertainty*

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October 29, 2024

Abstract

Endogenous uncertainty acts as an aggregate-demand amplification mechanism of supply shocks. Using U.S. data, we first stress that taking into account time-varying macroeconomic uncertainty leads to a significantly stronger recession and less inflationary pressures, in response to a TFP shock. In addition, we show empirically that households' misperception increases during recessions. To rationalize these findings, we build a noisy-information New-Keynesian model where the precision of signals increases with economic activity. Pro-cyclical precision of information gives rise to an amplified precautionary saving behavior. A full-fledged model parametrized by using consumer-based forecast errors generates a demand-like recession of supply shock.

JEL classification: D81, D83, E21, E32.

Keywords: Uncertainty, imperfect information, Keynesian supply shocks.

*We thank Isaac Baley for useful comments. We acknowledge financial support from the French government under the "France 2030" investment plan managed by the French National Research Agency Grant ANR-17-EURE-0020, and by the Excellence Initiative of Aix-Marseille University - A*MIDEX. It was also supported by French National Research Agency Grant ANR-20-CE26-DEMUR. Céline Poilly thanks the Institut Universitaire de France for its financial support.

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1 Introduction

Global economies have recently experienced major economic upheavals – such as the COVID crisis in 2020 or the 2021–2023 inflation surge – which has led to a growing interest in understanding of the nature of economic shocks. For instance, a popular idea that has been renewed recently is that negative supply shocks might generate demand-like recessions (see among others Guerrieri et al. (2022)). The basic idea is that a recessionary supply shock, that reduces potential output, generates an endogenous substantial drop in aggregate demand which drives actual output *below* its potential level. This is commonly referred to as a "Keynesian supply shock". These demand-driven amplification effects are typically absent in a traditional New-Keynesian model since the sticky-price output under-reacts to a negative supply shock, relative to its flexible-price counterpart. Existing research has identified several mechanisms underlying the Keynesian supply effect, based, for instance, on the production-side reactions to shocks (Guerrieri et al. (2022), Bilbiie and Melitz (2023), Fornaro and Wolf (2023)) or the precautionary-saving motive of consumers (Ravn and Sterk (2017, 2021), Challe (2020)).

This paper brings an alternative rationale for explaining the aggregate-demand amplification of supply shocks. We argue that the *pro-cyclical* precision of information received by consumers magnifies the recessionary effects of a negative supply shock because it reinforces the precautionary motive of savers, which ultimately leads to negative demand effects.[1] Intuitively, recessions are usually characterized by a surge in consumer uncertainty, which can be interpreted as a rise in the degree of agents' misperceptions about the true state of the economy. Therefore, a negative supply shock not only reduces the economy's capacity to produce goods but also makes consumers more uncertain. They are thus encouraged to build precautionary savings, which causes a sizable drop in aggregate demand. Even though we are not the first to highlight the role of precautionary-saving behavior in the transmission of supply shocks, our contribution is to rationalize this mechanism through household-based information frictions that provoke endogenous uncertainty.[2]

We check the empirical validity of our intuition in two steps. First, we quantify the role of consumer uncertainty in the transmission of productivity shocks in the U.S. using the counterfactual scenario analysis methodology, in the spirit of Antolin-Diaz et al. (2021). Precisely, we

[1]The implications of pro-cyclical signal precision have previously been studied in the context of the Real Business Cycle (RBC) model by Van Nieuwerburgh and Veldkamp (2006), with a focus on the cyclical speed of learning, and by Fajgelbaum et al. (2017) and Ilut and Saijo (2021), who examined the implications of endogenous consumer confidence for firm investment decisions and economic activity. In contrast, we focus on the New-Keynesian aggregate demand amplification arising from the precautionary-saving channel.

[2]Ravn and Sterk (2017, 2021) and Challe (2020) analyze the aggregate-demand amplification through the precautionary-saving channel. However, they focus on counter-cyclical unemployment probability as a source of uninsured idiosyncratic risk for households in a HANK-SAM model. In contrast, we concentrate on uncertainty that arises from imperfect aggregate information within a representative agent New-Keynesian model.

estimate a Structural Vector Auto-Regressive (SVAR) model for the U.S. economy, composed of Total Factor Productivity (TFP), uncertainty perceived by consumers, and a set of other macroeconomic variables. We show that uncertainty rises after a negative supply shock, identified using a recursive identification strategy on TFP. Then, we build a counterfactual scenario in which the response of uncertainty to the supply shock is kept at zero. This allows us to quantify the extent of the recession that is explained by uncertainty-driven dynamics. We find that GDP drops by significantly more to the negative supply shock when consumer uncertainty is allowed to move. We also document that the endogenous evolution of uncertainty leads to a more pronounced adjustment in hours worked and exerts additional deflationary pressure on the economy following the shock. These empirical findings lead us to conclude that endogenous consumer uncertainty magnifies the recessionary effects of negative supply shocks.

In the second step, we test whether the fact that consumer uncertainty increases during recessions is due to a deterioration in the quality of information available to consumers. To do so, we use the Michigan Survey of Consumers (MSC) to build household-level forecast errors about their income growth. We then aggregate these errors to explore the persistence of the aggregate forecast error series. Under full (perfect) information, agents update their information sets by exploiting all available data, making forecast errors theoretically unpredictable from publicly available information. Following, for instance, Andrade and Le Bihan (2013), we interpret a rise in the persistence of forecast errors (i.e., predictability from previous forecast errors) as an increase in the degree of imperfect information. By estimating a simple auto-regressive process on our measure of consumers' forecast errors, and interacting it with a recession dummy, we confirm that the persistence of forecast errors – and therefore the degree of consumer misperceptions – is significantly stronger during recessions.

We then rationalize our findings by building a theoretical New-Keynesian model that incorporates pro-cyclical quality of information, accounting for endogenous consumer uncertainty. Then, we use this non-linear setup to analyze the role of precautionary-saving behavior in the transmission of supply shocks. Our model builds on a typical noisy information framework, where we assume that households receive a noisy signal about the true state of the economy, and that the precision of the signal is pro-cyclical, i.e. the uncertainty faced by consumers increases during economic downturns. We interpret this time-varying precision of signals as a "learning-by-doing" signal; in other words, consumers/workers, by being apart from the production side during recessions, are less informed about the state of the economy.[3] In total, the precision of

[3]Our information flow structure is somewhat similar to that of Fajgelbaum et al. (2017), with the key difference being that we motivate the pro-cyclical information flow through a "learning-by-doing" mechanism, where activity directly generates information. In contrast, Fajgelbaum et al. (2017) emphasize "social learning", where information is generated by observing the actions of peers. Foster and Rosenzweig (1995) show that both types of learning are empirically plausible. Note that, in the business cycle context, both mechanisms have similar implications – the pro-cyclical quality of available information.

this "learning-by-doing" signal depends on the level of production, which itself is influenced by the precision of the received signal. This interdependence lies at the core of our endogenous uncertainty amplification mechanism.

To build our intuition regarding the aggregate-demand amplification of supply shocks, we use a tractable version of our non-linear model and derive a risk-adjusted version of the Euler equation that accounts for the precautionary-saving motive. We then assess analytically the effect of TFP shock on output, inflation, and the output gap in a model with and without endogenous uncertainty. As is standard in a New-Keynesian model, under constant uncertainty, a negative TFP shock reduces potential output, while the output gap *increases* – due to a limited reduction in actual output – and inflation goes up. This result can be reversed when the precision of consumers' signals is pro-cyclical. Intuitively, a supply-driven recession makes consumers less informed about the state of the economy, which raises the aggregate uncertainty they face and encourages them to adopt precautionary-saving behavior. This generates a demand-driven recession in response to a negative productivity shock, which ultimately leads to a *reduction* in the output gap, a decrease in the price level, and a decline in hours worked.

In the last step, we proceed to a quantitative analysis of our endogenous uncertainty mechanism by using a full-fledged non-linear New-Keynesian model, which is parameterized on U.S. data using the aggregate forecast error series about income growth built in the empirical part. In particular, we set the precision and sensitivity of signals to output so as to match the percentiles of the aggregate forecast error series. In line with our intuition and theoretical results, we find that pro-cyclical precision of signals magnifies the supply-driven recession and is strong enough to make the negative supply shock deflationary. Taking a closer look at the parameters that drive the strength of this endogenous uncertainty channel, we find that the degree of risk aversion (as it affects the strength of the precautionary saving channel) or the degree of persistence of the supply shock (as it affects the amplitude of the drop in expected future income) matter for the dynamics.

Finally, we emphasize that procyclical precision of signals not only affect the transmission channels of supply shocks but also those of demand shocks, like public spending shocks. Indeed, it can reverse the typical crowding-out effect on private consumption after an expansionary public spending shock. The usual channel in the literature is that the rise in expected future taxes generates a negative wealth effect and thus a reduction in private consumption. Under endogenous uncertainty, the expansion reduces macroeconomic uncertainty, which dampens the precautionary-saving behavior of consumers and therefore limits the crowding-out effect on consumption. This effect depends on the degree of persistence of the shock and the reaction of the central bank.

Contributions to the literature

Our paper contributes to several strands of the literature. First, we relate to the literature on Keynesian supply shocks – supply-side disturbances that endogenously generate demand effects. The existing research identifies several alternative mechanisms underlying the Keynesian supply effect. For example, Fornaro and Wolf (2023) demonstrate that productivity-enhancing investment within the New-Keynesian model can lead to the Keynesian supply effect. Similarly, Cesa-Bianchi and Ferrero (2021) and Guerrieri et al. (2022) show that sectoral productivity shocks can trigger strong aggregate demand effects in a multi-sectoral economy; Bilbiie and Melitz (2020) explore the aggregate demand amplification through the firm entry–exit multiplier. L'Huillier et al. (2024) find that Keynesian supply shocks can emerge in the New-Keynesian model when incorporating non-rational (diagnostic) expectations. We contribute to this literature by analyzing an alternative mechanism for aggregate demand amplification, which is based on the feedback loop between uncertainty and economic activity operating through the precautionary saving channel. Unlike mechanisms outlined above, ours does not rely on production-side features but instead, is grounded in the idea that endogenous fluctuations in consumer uncertainty influence consumption demand, ultimately impacting the level of economic activity. To analyze this mechanism, we extend the standard New-Keynesian noisy information framework of Woodford (2001) or Lorenzoni (2009) by incorporating pro-cyclical information quality, which results in endogenous uncertainty.[4]

The demand-side amplification through the precautionary saving channel, driven by time-varying, endogenous uncertainty, makes our paper conceptually similar to the amplification mechanism in the HANK&SAM models of Ravn and Sterk (2017, 2021) and Challe (2020). However, the difference is that in our model, endogenous uncertainty arises from informational friction rather than labor market imperfection. As a result, uncertainty in our model is aggregate rather than idiosyncratic, and it triggers precautionary saving even under full risk sharing across households, making redistributive policies, such as insurance or transfers, ineffective.[5]

Second, our paper relates to the literature on imperfect information and learning. Existing empirical works testing the Full Information Rational Expectations (FIRE) hypothesis Mankiw et al. (2003); Andrade and Le Bihan (2013) provide strong evidence against it, while Coibion and Gorodnichenko (2012, 2015) develop an empirical test to further distinguish between various imperfect information models. Building on this literature, we employ the econometric test of FIRE, which relies on the persistence of expectation errors, but we extend this test to explore the

[4]See also Bomfim (2001), who introduces noisy information into a standard RBC model.

[5]Studies examining the effect of *exogenous* aggregate uncertainty fluctuations on precautionary saving include Fernández-Villaverde et al. (2011), Leduc and Liu (2016), Basu and Bundick (2017), Fernández-Villaverde and Guerrón-Quintana (2020) among others. In contrast to these studies, the evolution of uncertainty in our model is *endogenous* and driven by shocks to fundamentals, which lead to variations in information precision due to informational frictions.

cyclical dimension of information imperfections.

Within the imperfect information literature, our paper is related to research on pro-cyclical learning in a noisy information environment, as explored by Veldkamp (2005); Van Nieuwerburgh and Veldkamp (2006); Ordonez et al. (2009); Mäkinen and Ohl (2015). In particular, it is related to studies examining the implications of endogenous uncertainty in business cycle models. These studies consider various dimensions of uncertainty propagation, with a focus on investment (Saijo, 2017; Fajgelbaum et al., 2017; Schaal and Taschereau-Dumouchel, 2023), Knightian uncertainty (Ilut and Saijo, 2021), financial frictions (Benhabib et al., 2019; Straub and Ulbricht, 2024) or labor market frictions (Bernstein et al., 2024). In contrast to this literature, we investigate the implications of endogenous *consumer* uncertainty for aggregate shock propagation within a noisy information New-Keynesian model, with a focus on the precautionary saving channel.

2 Motivating Evidence

In this section, we first assess the contribution of consumer uncertainty to the transmission of productivity shocks in the U.S.. Second, we check that information frictions are stronger during recessions. These two pieces of empirical evidence motivate our theoretical framework, which features endogenous uncertainty stemming from the pro-cyclical fluctuations in the quality of information received by consumers.

2.1 Amplification through consumer uncertainty

We examine the role of consumer uncertainty in the transmission of productivity shocks to economic activity using the structural scenario analysis methodology laid out in Antolin-Diaz et al. (2021).[6] First, we estimate a SVAR model in which we identify a productivity shock, allowing us to assess the impact of this shock on both consumer uncertainty and overall economic activity. Then, we construct a counterfactual impulse response in which consumer uncertainty is held constant. This is achieved by introducing a set of counterfactual uncertainty shocks that neutralize the impact of the productivity shock on uncertainty over time.

Methodology. We now describe the methodology, following the notation of Antolin-Diaz et al. (2021). Consider a SVAR model of the form $y_t'A_0 = x_t'A_+ + \epsilon_t$ where $x_t' = [y_t', \dots\ y_{t-p}',\ 1]$, $A_+ = [A_1', \dots\ A_p',\ d']$, and p is the number of lags. Let n denote the number of variables in the model. The vector of structural shocks ϵ_t is distributed as $\mathcal{N}(\mathbf{0}_{n\times 1}, I_n)$. The corresponding reduced form VAR is given by $y_t' = x_t'B + v_t$ where $E[v_t v_t'] = \Sigma = A_0 QQ' A_0'$, with Q being the

[6] See also McKay and Wolf (2023) for counterfactual construction based on news shocks, and Georgiadis et al. (2024) for its application to global risk shocks.

orthogonal rotation matrix implied by the identifying restrictions, as discussed Rubio-Ramirez et al. (2010).

We identify a productivity shock ϵ_t^a and an uncertainty shock ϵ_t^u using zero restrictions. Specifically, the productivity shock is identified as the only shock that has a contemporaneous effect on the productivity variable a_t, while the uncertainty shock is identified as the only shock with a contemporaneous effect on the uncertainty variable u_t. Although our focus is on analyzing the effect of the productivity shock, the identified uncertainty shock is used to construct a counterfactual scenario in which uncertainty remains constant.

Given a sequence of shock realizations over the forecast horizon h (from $T+1$ to $T+h$) $\epsilon'_{T+1,T+h} = [\epsilon'_{T+1}, .. \ \epsilon'_{T+h}]$, the unconditional forecast $y'_{T+1,T+h} = [y'_{T+1}, .. \ y'_{T+h}]$ is given by $y_{T+1,T+h} = b_{T+1,T+h} + M'\epsilon_{T+1,T+h}$ where $b_{T+1,T+h}$ is the path predetermined by the history preceding T, and M' is a matrix of structural parameters, i.e. it depends on the identification restrictions in Q. Notice that $M'\epsilon_{T+1,T+h}$ captures the impulse response to an arbitrary sequence of shocks. In this paper, the unconditional impulse response to the productivity shock is constructed by setting $\epsilon^a_{T+1} = 1$, and $\epsilon^a_{T+k} = 0$ for all $k > 1$ and $\epsilon^s_{T+k} = 0$ for all $s \neq a$ and all $k > 0$.

Let $\tilde{y}_{T+1,T+h}$ denote the counterfactual (or conditional) forecast that is obtained by picking up a counterfactual series of shocks $\tilde{\epsilon}_{T+1,T+h}$. In our exercise, $\tilde{\epsilon}_{T+1,T+h}$ aims at simulating the effect of technology shocks under constant uncertainty. In practice, the counterfactual series of shocks $\tilde{\epsilon}_{T+1,T+h}$ satisfies the condition $C\tilde{y}_{T+1,T+h} = Cb_{T+1,T+h} + CM'\tilde{\epsilon}_{T+1,T+h}$, where C is a $(h \times nh)$ restriction matrix that selects the response of uncertainty to the technology shock and ensures that it remains constant over the forecast horizon h, i.e. $C\tilde{y}_{T+1,T+h} = 0$ for all $k > 0$. We restrict the set of counterfactual shocks to include only uncertainty shocks $\tilde{\epsilon}^u_{T+k}$ $(0 < k \leq h)$, which are computed to offset the impact of productivity shock ϵ^a_{T+1} on uncertainty over the forecast horizon h.[7]

Data. We estimate a quarterly SVAR model for the U.S. using data from 1981Q1 to 2019Q4. As a productivity measure, we employ the utilization-adjusted TFP series from Fernald (2012). Consumer uncertainty is measured using a self-reported uncertainty indicator from the MSC. This uncertainty measure is based on consumer sentiment and is calculated as the share of respondents who cite an "uncertain future" as the reasons for not purchasing large household durables. The baseline model also includes real consumption, hours worked, real GDP, the consumer price index, real wages, real investment, stock prices, business formation, nominal interest rates, and consumption for durable goods.[8] We estimate the SVAR model with $p = 5$ lags and consider a forecast horizon of $h = 40$ quarters when constructing the counterfactual scenario. In

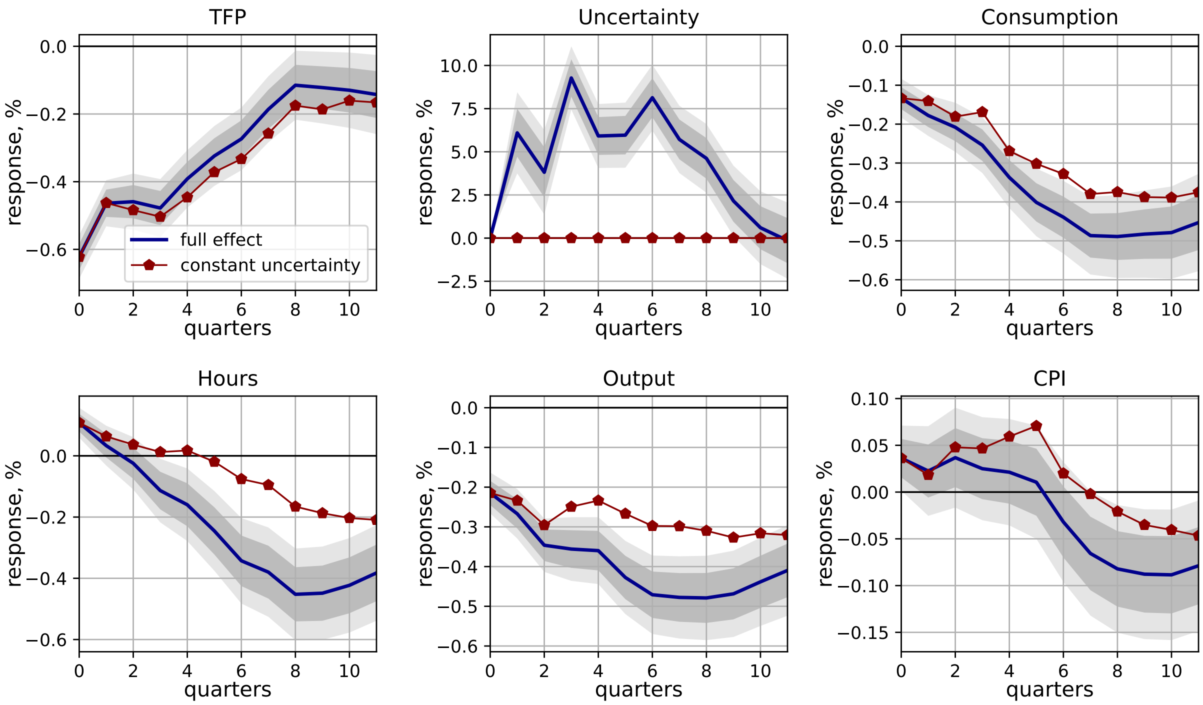
[7] As explained by Antolin-Diaz et al. (2021), if the set of counterfactual shocks is unrestricted there might be multiple ways to choose these shocks, and a specific choice procedure, such as Frobenius norm minimization, should be used. By restricting the type of shock we ensure that no such multiple possibilities exist.

[8] Data description is provided in Table A.1.

the online appendix, we show that our results are invariant to the scale of the SVAR model.

Results. Figure 1 shows the impulse response functions (IRFs) of some selected variables to a negative productivity shock, along with the corresponding counterfactual responses where uncertainty is held constant.

Figure 1: Unrestricted and restricted responses to a negative productivity shock



Note: The blue solid lines correspond to the response of the variables when uncertainty is unrestricted. The red dotted lines show the response in the counterfactual scenario where uncertainty is held constant. The shaded areas indicate the 68% and 90% bootstrapped confidence bands.

The solid lines illustrate that the negative productivity shock is recessionary, leading to declines in both consumption and GDP. Consumer prices show a modest initial rise in the short-run – confirming that we identify a supply shock. Hours worked experience a slight increase on impact but then decline in the subsequent periods. Importantly, consumer uncertainty spikes in response to the negative productivity shock. In contrast, the counterfactual scenario displayed by dotted lines – where uncertainty's response remains constant – shows a significantly milder recession. Indeed, the declines in consumption, GDP, and hours worked are significantly less severe. Moreover, prices experience a more pronounced initial increase with a far weaker subsequent decline, suggesting that the supply shock is more inflationary under constant uncertainty. This comparison suggests that the increase in consumer uncertainty significantly amplifies the recessionary effects of a productivity shock.

2.2 Pro-cyclical flow of information

In the previous subsection, we documented that the endogenous rise in consumer uncertainty drives a significant part of the recessionary effects resulting from a negative productivity shock. In this paper, we argue that the surge in uncertainty during a recession can be rationalized through a deterioration in the quality of information, i.e. the precision of the signals received by agents is supposed to decline. To check the validity of our intuition, we construct a monthly series of aggregate income growth forecast errors for the U.S. and we examine its behavior during recession. Precisely, we resort to the U.S. household-level database from the MSC.[9] In each interview, respondents are asked to report their current household income (in dollars) as well as their expected income growth over the next 12 months.[10] For each month, we consider only respondents who have been re-interviewed at least once. Our sample spans from January 1981 to December 2019.

Let $\mathbb{E}_{t-12}[\Delta inc_{j,t}]$ denote the expected income growth reported by the j-th respondent during her first interview, referring to the income growth she expects to receive over the upcoming year. Using the data from both interviews, we can also calculate $\Delta inc_{j,t}$, the realized income growth over the same year for the j-th respondent, where $j \in \{1, \ldots, J\}$. For each month, we compute the individual forecast errors and then aggregate them across individuals to obtain a monthly series of the aggregate income growth forecast errors, $e^f_{t,t-12}$, calculated as $e^f_{t,t-12} = \Sigma^J_{j=1}\omega_{j,t}\left(\mathbb{E}_{t-12}[\Delta inc_{j,t}] - \Delta inc_{j,t}\right)$, where $\omega_{j,t}$ is the weight assigned to the j-th respondent's forecast error at time t ($\Sigma^J_{j=1}\omega_{j,t} = 1$) and provided by the MSC.

To test for time-varying information precision, we explore the serial correlation of forecast errors, a common approach in the literature on information imperfections (Mankiw et al., 2003; Andrade and Le Bihan, 2013). This approach is based on the martingale property of FIRE models, which implies that ex-post forecast errors should not be predictable based on the available set of observable information (Pesaran and Weale, 2006). Intuitively, under full (perfect) information, agents are supposed to update their set of information immediately and without any cost, implying that the ex-post forecast errors are nil on average and not predictable from the available data. On the contrary, finding that forecast errors are predictable from observable variables,

[9] The Michigan Survey of Consumers is a rotating monthly panel survey in which respondents are eligible to be re-interviewed six months after the initial interview. As explained by the Surveys of Consumers Technical Report (2024), "an independent cross-sectional sample is drawn each month, and those who completed interviews in a given month become eligible for re-interviews approximately six and twelve months later. Thus, each monthly sample is composed of a mix of interviews from the independent cross-sectional sample and the recontact sample."

[10] The current income corresponds to 'INCOME: total household income - current dollars' (the asked question is "*Now, thinking about your total income from all sources (including your job), how much did you receive in the previous year?*") and the expected income is 'FAMILY INCOME % u/d next year' (the asked question is "*By about what percent do you expect your income to (increase/decrease) during the next 12 months?*")

like for instance previous forecast errors, goes in favor of imperfect information.[11] Following this literature, we investigate whether ex-post forecast errors are serially correlated and whether this persistence increases during recessions, that would suggest that imperfect information issue worsens during economics downturns.[12]

We assess the time-varying persistence of the income growth forecast errors, e^f_{t-12}, by regressing the forecast errors on its lag and its lag interacted with a recession dummy

$$e^f_{t,t-12} = \beta_0 + \beta_1 \cdot e^f_{t-13,t-24} + \beta_2 \cdot e^f_{t-13,t-24} \cdot rec_{t-13} + \sum_i \beta_i \cdot X_{i,t-13} + v_t, \tag{1}$$

where rec_{t-13} is a dummy variable taking one during a recession period (as defined by the NBER Business Cycle Dating), $X_{i,t-13}$ is the i^{th} control variable, and v_t is the error term. Note that the lag constitutes 12 months, as our forecast errors concern the yearly income growth, and the realized error from the previous year should be available to consumers when making forecasts about future income.

The first column of Table 1 presents the results from regressing forecast errors on their lagged values and a constant, which is a standard econometric test of imperfect information. A significant coefficient of serial correlation suggests that some information from the forecast error a year ago has not been incorporated into the current income growth forecast, which contradicts the FIRE assumptions. Column (2) in Table 1 reports the results of the extended regression that aims to test the change in error persistence during recessions. This regression introduces an interaction term between the previous forecast error and a recession dummy as an additional explanatory variable. The estimate of the unconditional autocorrelation coefficient, β_1, remains significant, indicating that the persistence of forecast errors is not solely driven by recession periods. The positive and significant coefficient on the interaction term, β_2, indicates that the persistence of forecast errors roughly doubles during recessions, increasing from approximately 0.29 to 0.59. Through the lens of the noisy information model, this evidence suggests that information tends to be less precise during recessions, resulting in greater error persistence. Finally, we extend our specification by incorporating various macroeconomic indicators as additional explanatory variables. In Column (3), we present our estimates of persistence while controlling for aggregate

[11] Angeletos et al. (2021) point out that the persistence of errors may stem from "old-fashioned" adaptive expectations, which is a particular type of non-rational expectations. However, the data clearly reject the adaptive expectations model (see, for example, Ball (2000) and Mankiw et al. (2003)). See also Coibion and Gorodnichenko (2015) who focus on the properties of the rational expectations models with imperfect information.

[12] While the literature typically employs forecast errors regarding aggregate variables, we construct our aggregate forecast error from the underlying idiosyncratic income growth data. As a result, our aggregate forecast error pertains to the aggregate component of income growth. In Appendix B we provide a stylized dynamic income model with noisy information and Bayesian learning and show that within this model aggregation removes the idiosyncratic income growth component. As a result, the standard econometric test of forecast error persistence can also be applied to the aggregate income growth forecast error series. In Appendix B, we also show that aggregate forecast error persistence should increase in recessions provided that the quality of information is procyclical.

Table 1: Persistence of forecast errors

	Dependent variable: Forecast error $e^f_{t,t-12}$		
	(1)	(2)	(3)
β_1: $e^f_{t-13,t-24}$	0.330***	0.289***	0.272***
	(0.043)	(0.044)	(0.046)
β_2: $e^f_{t-13,t-24} \cap rec_{t-13}$		0.305***	0.338***
		(0.093)	(0.094)
β_3: $Income\ growth_{t-13}$			−0.011
			(0.052)
β_4: $Inflation_{t-13}$			0.763
			(0.685)
β_5: $Unemployment_{t-13}$			−0.223**
			(0.111)
β_0: Constant	2.317***	2.306***	3.587*
	(0.242)	(0.240)	(0.778)
Observations	456	456	455
Adjusted R^2	0.113	0.131	0.134

Note: Recession corresponds to NBER recession dates. The unemployment rate is the share of unemployed in the labor force, expressed as a percentage and seasonally adjusted. Inflation is calculated from the CPI for All Urban Consumers, seasonally adjusted. Both data series are from the U.S. Bureau of Labor Statistics. *p<0.1; **p<0.05; ***p<0.01.

income growth, inflation, and the unemployment rate. Although the explanatory power of these variables, in and of itself, violates the FIRE hypothesis, we include them primarily to control for potential confounding effects. Our results show that the coefficient on the interaction between lagged forecast errors and the recession dummy (β_2) remains large and significant despite the inclusion of other potential forecast error predictors.[13]

This section led us to conclude that consumers update their information set less easily during recessions, those recessions being characterized by endogenous spikes in uncertainty. We now build a theoretical model that rationalizes endogenous uncertainty through fluctuations in the quality of information. We then examine how endogenous uncertainty affects the transmission channels of productivity shocks.

[13] We also experimented with different macroeconomic controls and considered adding additional lags, but found that our main result remained robust to these extensions.

3 A New-Keynesian Model with Endogenous Uncertainty

We base our model on the noisy information New-Keynesian framework, in the spirit of Woodford (2001) and Lorenzoni (2009), which we extend in two dimensions. First, drawing on the widely accepted idea that economic activity generates information, we introduce pro-cyclical signal precision, leading to endogenous, time-varying uncertainty. Second, we account for household precautionary-saving behavior by departing from the linear framework. These two extensions give rise to a novel endogenous uncertainty channel for aggregate shock propagation, driven by the feedback loop between consumer uncertainty and economic activity – a mechanism absent in the standard noisy information New-Keynesian model. We present our model in two steps. First, we lay out the features that are standard elements of the New-Keynesian model. Then, we describe the information structure and belief updating with pro-cyclical precision.

3.1 Basic features

A representative household consumes, saves, and supplies labor. A continuum of monopolistically competitive firms uses labor as the only input to produce intermediate goods, facing Rotemberg adjustment cost when setting prices. These intermediate goods are then aggregated into a final consumption good purchased by the household and the government. The central bank follows a Taylor rule, while the government finances its spending through a lump-sum tax. Fluctuations are driven by aggregate productivity shocks.

3.1.1 Household

A representative household chooses consumption, saving, and labor to maximize its expected lifetime utility

$$\max_{C_t,\, L_t,\, B_t,\, b_t} \quad \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\eta}}{1-\eta} - \frac{L_t^{1+\omega}}{1+\omega} \right], \tag{2}$$

where $\mathrm{E}_0\{\cdot\}$ is the expectation operator, C_t denotes the household's consumption of the final good, and L_t is the labor supply (hours worked). The parameter η measures the household's relative aversion to risk, ω is the inverse Frisch elasticity of labor supply, and β is the discount factor. The household lifetime utility in Eq. (2) is subject to the following sequence of budget constraints $P_tC_t + B_t + P_tb_t = R_{t-1}B_{t-1} + r_{t-1}P_{t-1}b_{t-1} + P_tW_tL_t + D_t$. In this budget constraint, P_t denotes the price level in the period t, and W_t is real wage. The household saves into two saving vehicles: nominal and real one-period riskless bonds, denoted by B_t and b_t, respectively. Both types of bonds are in zero-net supply and provide a gross nominal interest rate R_t and a real interest rate r_t, respectively. The inclusion of both real and nominal bonds keeps the model general enough, enabling us to investigate the precautionary-saving mechanism in Section 4 while abstracting from the inflation risk premium created by inflation expectations. The household also receives a nominal payment D_t, which consists of dividends from firm ownership and net government transfers. The household's optimization yields two Euler equations and the labor supply equation

$$1 = \beta(1 + r_t)\mathrm{E_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\eta} \quad or \quad 1 = \beta\mathrm{E_t}\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\eta}\frac{R_t}{\pi_{t+1}}\right\}, \tag{3}$$

$$L_t^{\omega} = C_t^{-\eta}W_t, \tag{4}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate. We also define the stochastic discount factor as $Q_{t,t+1} \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\eta}\frac{1}{\pi_{t+1}}$. Notice that in the stylized model of Section 4, we will resort to the first expression in Eq. (3) while we will use the second and thus the more general expression in the quantitative illustration, see Section 5. The reason is that the first expression allows us to concentrate on the precautionary-saving motive as it excludes inflation expectations.

3.1.2 Firms

Representative final good firms operate in a competitive market and produce final output Y_t, by using a bundle of differentiated intermediate goods purchased from a continuum of monopolisitcally competitive firms, such that $Y_t(i)$ is purchased from the i-th firm at price $P_t(i)$. The final good Y_t is produced according to a CES technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$, where ϵ is the elasticity of substitution between differentiated goods. Thus, the demand for differentiated goods is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}Y_t, \tag{5}$$

and the aggregate price index is $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}\mathrm{d}i\right)^{\frac{1}{1-\epsilon}}$.

A unit mass of monopolistically competitive firms, indexed by $i \in [0,1]$, each produce a differentiated good $Y_t(i)$, using the following production function

$$Y_t(i) = \tilde{A}_t L_t(i)^{1-\alpha}, \tag{6}$$

where $\tilde{A}_t$ drives the aggregate productivity (i.e. TFP) process, which will be described in detail below, and $(1-\alpha)$ denotes the returns to scale.

Each monopolistically competitive firm i acts as a price-setter by choosing the price of its differentiated good, $P_t(i)$, while facing Rotemberg (1982) nominal quadratic costs of price adjustment, given by $\Phi_t(i) \equiv \frac{\Phi}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 P_t Y_t$, where Φ determines the degree of price rigidity.

Each firm maximizes its expected discounted stream of future profits

$$\max_{P_t(i),Y_t(i)} \sum_{t=0}^{\infty} Q_{0,t} \left[P_t(i)Y_t(i) - (1-\bar{\tau})P_tW_tL_t - \frac{\Phi}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 P_tY_t \right], \tag{7}$$

subject to the sequence of firm-specific demand given by Eq. (5). Here $\bar{\tau} = \epsilon^{-1}$ is a standard labor subsidy ensuring that ensures the efficiency of the flexible-price equilibrium. The first-order condition for profit maximization, evaluated in a symmetric equilibrium, yields the conventional New-Keynesian Phillips Curve

$$\epsilon\left(1-(1-\bar{\tau})MC_t\right) = 1 - \Phi\left(\pi_t - 1\right)\pi_t + \Phi \mathrm{E}_t\left\{Q_{t,t+1}\left(\pi_{t+1}-1\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right\}, \tag{8}$$

where MC_t denotes the real marginal cost. The labor demand equation, derived from the firms' cost-minimization problem and evaluated at the symmetric equilibrium, is given by:

$$W_t = (1-\alpha)\,\tilde{A}_t MC_t L_t^{-\alpha}. \tag{9}$$

3.1.3 Policy and resource constraint

The central bank sets the nominal interest rate R_t according to the Taylor (1993) rule

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}, \tag{10}$$

where $\bar{Y}$ is the steady-state output and $\bar{R} = 1/\beta$ is the steady-state nominal interest rate (with $\bar{\pi} = 1$ in the stable-price steady state). The parameter ϕ_π determines the sensitivity of the interest rate to inflation, while ϕ_y governs its sensitivity to output. Finally, there is an exogenous stream of government spending G_t financed through a lump-sum tax.[14] The aggregate resource constraint is given by

$$Y_t = C_t + G_t + \frac{\Phi}{2}(\pi_t - 1)^2 Y_t. \tag{11}$$

3.1.4 Productivity

Productivity, $\tilde{a}_t \equiv \log(\tilde{A}_t)$, consists of two components: a persistent component a_t and a transitory component f_t, such that

$$\tilde{a}_t = a_t + f_t, \tag{12}$$

[14] In this paper, we abstract from randomness regarding G_t by simulating the model under a deterministic transition path of government expenditure, see Section 5 for details.

where f_t is a white noise process with $f_t \sim \mathcal{N}(0, \sigma_f^2)$ and a_t follows an AR(1) process

$$a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \epsilon_t^a, \tag{13}$$

where ρ_a is the persistence of a_t, and the innovation is normally distributed $\epsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$. The two shocks, f_t and ϵ_t^a are mutually independent. We depart from the typical full information model by assuming that agents observe the current level of productivity, $\tilde{a}_t$, but they *cannot* disentangle between the transitory component f_t and the persistent component a_t, which is a common assumption in the noisy information New-Keynesian literature (Lorenzoni, 2009).

3.2 Information structure

While the true state of the economy, that is the persistent productivity component a_t, is unknown to agents, they receive noisy signals about it. At the beginning of each period t, agents hold a prior belief regarding a_t. During the period, agents update their beliefs using noisy information obtained from two sources: (1) the observed realized productivity $\tilde{a}_t$, and (2) an additional "learning-by-doing" signal with time-varying precision. The precision of the time-varying signal is pro-cyclical, meaning it fluctuates with the state of the economy and increases during periods of economic expansion. This pro-cyclical pattern is motivated by the idea that economic activity generates information. Based on their prior beliefs and the information contained in the signals they receive, agents make decisions regarding consumption, labor supply, and production, while simultaneously updating their beliefs in a Bayesian manner. The precision of the "learning-by-doing" signal depends on the level of production, which itself is influenced by the precision of the received signal. This interdependence lies at the core of our endogenous uncertainty amplification mechanism. We next provide a detailed description of the belief formation process.

3.2.1 Priors

Households begin period t with prior beliefs about persistent productivity. Let Ω_t denote the information set available to households at time t before they receive any signals pertaining to the *current* period. Given this information, the prior belief about the persistent productivity component is

$$a_t | \Omega_t \sim \mathcal{N}(\theta_t, \gamma_t^{-1}). \tag{14}$$

where θ_t is the perceived mean, and γ_t^{-1} is the perceived variance (γ_t is the precision of the available information that is used to construct the prior belief). Notice that prior beliefs about productivity are therefore state variables in our model, as they are predetermined at time t.

During period t, after forming their priors, agents receive two noisy signals. The first signal

z_t amounts to observing the productivity level $\tilde{a}_t$ and has constant precision.[15] The second signal, s_t, is a "learning-by-doing" type of signal with procyclical precision. Agents use the information from both z_t and s_t to update their prior beliefs and determine the next period's prior belief θ_{t+1} and γ_{t+1} via Bayesian learning, which we describe below.

3.2.2 Signals

The first signal that agents receive, z_t, amounts to observing the productivity $z_t = \tilde{a}_t$. Given this signal, the belief about the persistent productivity component is given by $a_t|z_t \sim \mathcal{N}(\tilde{a}_t, [\gamma^z]^{-1})$, where the constant precision of this signal is $\gamma^z = \sigma_f^{-2}$. From Eq. (12), we can express the noisy signal z_t as

$$z_t = a_t + \epsilon_t^z \tag{15}$$

where $f_t = \epsilon_t^z \sim \mathcal{N}(0, [\gamma^z]^{-1})$ corresponds to a noise shock.

The second signal that agents receive regarding the persistent component a_t is a noisy "learning-by-doing" signal, denoted by s_t, with procyclical precision. It is defined as

$$s_t = a_t + \epsilon_t^s, \tag{16}$$

where $\epsilon_t^s \sim \mathcal{N}(0, [\gamma_t^s]^{-1})$ is a noise shock . Our novelty is to assume that the precision of this signal is time-varying and increases with the level of economic activity, such that $\gamma_t^s = \gamma\left(Y_t/\bar{Y}\right)$, reflecting the "learning-by-doing" nature of this information source.[16] We, therefore, enrich the noisy information framework of Woodford (2001) and Lorenzoni (2009) by capturing the idea that the economic activity generates information enabling agents to form their expectations with greater precision, in the spirit of Van Nieuwerburgh and Veldkamp (2006); Fajgelbaum et al. (2017); Ilut and Saijo (2021), among others.[17]

[15]The signal z_t is introduced into the model primarily for interpretative purposes and is not essential to our main result. In our model, a_t is the unobserved persistent productivity component, while $\tilde{a}_t$ contains noisy information about this component. Thus, $\tilde{a}_t$ can be interpreted as a noisy signal, which we denote z_t.

[16]In Appendix C, we present alternative underlying learning protocols that result in the same pro-cyclical information precision behavior within the context of our model.

[17]Fajgelbaum et al. (2017) and Ilut and Saijo (2021) justify their modeling choices through the production-based information received by firms at a disaggregated level. Instead, in the spirit of Van Nieuwerburgh and Veldkamp (2006), we assume that the *aggregate* economic activity is the only source of information contained in the signal received by all agents.

3.2.3 Beliefs formation

Solving the signal extraction problem using Bayesian updating yields the agents' expectation of a_t based on the prior information Ω_t and two signals z_t and s_t

$$\mathrm{E}(a_t|\Omega_t, s_t, z_t) = \frac{\gamma_t\theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{17}$$

$$\mathrm{Var}(a_t|\Omega_t, s_t, z_t) = \frac{1}{\gamma_t + \gamma^z + \gamma_t^s}, \tag{18}$$

where $\Omega_{t+1} = \{\Omega_t, z_t, s_t\}$ is the set of information available at the end of the period. By combining the AR(1) productivity process in Eq. (13) with the mean belief in Eq. (17), we can derive the beliefs about a_{t+1} at the beginning of period $t+1$, denoted as $\theta_{t+1} \equiv \mathrm{E}(a_{t+1}|\Omega_{t+1})$

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a \mathrm{E}(a_t|\Omega_t, s_t, z_t) + \mathrm{E}(\epsilon_{t+1}^a|\Omega_{t+1}), \tag{19}$$

$$= (1-\rho_a)\bar{a} + \rho_a \frac{\gamma_t\theta_t + \gamma^z z_t + \gamma_t^s s_t}{\gamma_t + \gamma^z + \gamma_t^s}. \tag{20}$$

The precision of these beliefs, denoted as $\gamma_{t+1} \equiv [\mathrm{Var}(a_{t+1}|\Omega_{t+1})]^{-1}$, is obtained by combining Eq. (13) with Eq. (18)

$$\gamma_{t+1} = \left[\rho_a^2 \mathrm{Var}(a_t|\Omega_t, s_t, z_t) + (1-\rho_a)^2 \mathrm{Var}(\bar{a}) + \mathrm{Var}(\epsilon_t^a)\right]^{-1}, \tag{21}$$

$$= \left[\frac{\rho^2}{\gamma_t + \gamma^z + \gamma_t^s} + \sigma_a^2\right]^{-1}. \tag{22}$$

As a result, Eq. (20) and (22) jointly establish the recursive law of motion for the beliefs regarding the productivity component, a_t. In the next section, we explore in detail how these two processes affect the transmission channels of productivity shocks.

4 Understanding the Uncertainty Channel: A Stylized Approach

In this section, we demonstrate that the combination of counter-cyclical endogenous uncertainty – stemming from pro-cyclical information quality – and the precautionary-saving channel gives rise to a novel transmission mechanism absent in the standard New-Keynesian model: the endogenous uncertainty channel. To investigate our mechanism analytically, we rely on a log-linear approximation adjusted for the precautionary-saving motive. Specifically, in the spirit of Skinner (1988), we augment an otherwise linear Euler equation by an additional second-order term that captures precautionary-saving behavior. We now consider a fully tractable version of our baseline setup, obtained by making several simplifying assumptions.

First, we assume that nominal price rigidities apply only in the current period t. This implies that the inefficient wedge between the real wage and the marginal product of labor (measured by the real marginal cost) exists only in the current period and not in future periods. Second, we assume a linear production function ($\alpha = 0$), zero steady-state government spending ($\bar{G} = 0$), and that real bonds are the only saving vehicle in the economy, see first expression in Equation (3).[18] In terms of notation, we use lowercase letters to denote the logarithms of their corresponding uppercase variables, such that $x_t = log(X_t)$ unless specified otherwise. Additionally, we denote deviations from the steady state with $\hat{x}_t$. All derivations are provided in Appendix D.

Given our timing assumption, log-linearizing the labor demand Eq. (9) yields

$$w_{t+j} = mc_{t+j} + \tilde{a}_{t+j} \quad \text{with} \begin{cases} mc_{t+j} \neq 0 & \text{if} \quad j = 0 \\ mc_{t+j} = 0 & \text{if} \quad j > 0 \end{cases} . \tag{23}$$

By combining labor supply (4), labor demand (9), production function (6), and resource constraint (11), we obtain the following expression for output

$$y_t = \left(\frac{1}{\omega + \eta}\right) mc_t + \left(\frac{1+\omega}{\omega + \eta}\right) \tilde{a}_t + \left(\frac{\eta}{\omega + \eta}\right) g_t, \tag{24}$$

$$y_{t+j} = \left(\frac{1+\omega}{\omega + \eta}\right) \tilde{a}_{t+j} + \left(\frac{\eta}{\omega + \eta}\right) g_{t+j}, \quad \text{for } j > 0. \tag{25}$$

where $g_t = \frac{G_t}{\bar{Y}}$ is the ratio of government spending to steady-state output. We assume that government spending has persistence ρ_g, such that $g_{t+j} = \rho_g^j g_t$. From Eq. (25), we see that future output is fully determined by technology, as the absence of nominal rigidities from $t+1$ implies no future demand effects. In contrast, Eq. (24) indicates that present output depends on the endogenous wedge mc_t arising from nominal frictions. Lastly, for simplicity, we assume that there is no prior information about productivity, meaning $\gamma_t = 0$.[19]

4.1 IS curve and endogenous uncertainty

To understand our endogenous uncertainty channel, we derive a risk-adjusted IS curve that accounts for the precautionary-saving motive. This IS curve is obtained by combining the Euler equation (3) with the resource constraint (11). The detailed derivations can be found in Appendix D. The resulting risk-adjusted IS curve is

$$y_t = (1 - \rho_g) g_t - \frac{1}{\eta}(r_t - \rho) + \mathrm{E}_t\{y_{t+1}\} - \frac{1}{2}(1+\eta)\mathrm{Var}_t\{y_{t+1}\}, \tag{26}$$

[18] Assuming a linear production function and zero steady-state government spending simplifies the derivations but does not affect the analytical tractability of the model or any of our main theoretical results.

[19] Although this assumption makes beliefs more sensitive to new information, it does not affect our qualitative results. For a more general case allowing for an arbitrary value of γ_t, see Appendix D

where $\rho = -log(\beta)$. Eq. (26) sheds light on the aggregate demand side determinants of output dynamics. The first three terms correspond to a textbook New-Keynesian dynamic IS curve (Galí, 2008). The first term is a demand shifter driven by government spending, g_t. The second term reflects the negative link between output and the interest rate, as an increase in r_t makes it more desirable for households to trade part of their present consumption for future consumption. The third term captures the positive relationship between expected future output, $E_t\{y_{t+1}\}$ and current output. Importantly, our IS curve is augmented with a fourth term that reflects uncertainty about future output, expressed in terms of variance, $Var_t\{y_{t+1}\}$. This term illustrates the negative link between current output and uncertainty about future output, as higher uncertainty forces households to reduce their current consumption to accumulate precautionary savings.

From Eq. (26), it is clear that current output depends on *beliefs* about future output, specifically $\mathrm{E_t}\{y_{t+1}\}$ and $\mathrm{Var_t}\{y_{t+1}\}$. Our tractable setup enables us to compute these beliefs analytically in terms of the exogenous variables $\tilde{a}_t$ and g_t using Eq. (25)

$$\mathrm{E_t}\{y_{t+1}\} = \left(\frac{1+\omega}{\omega+\eta}\right)\mathrm{E_t}\{\tilde{a}_{t+1}\} + \rho_g\left(\frac{\eta}{\omega+\eta}\right)g_t \quad \text{and} \quad \mathrm{Var_t}\{y_{t+1}\} = \left(\frac{1+\omega}{\omega+\eta}\right)^2\mathrm{Var_t}\{\tilde{a}_{t+1}\}. \tag{27}$$

Here, the expression for variance relies on the assumption that government spending is deterministic. Note that $\mathrm{E_t}\{\tilde{a}_{t+1}\} = \mathrm{E_t}\{a_{t+1}\} = \theta_{t+1}$ and $\mathrm{Var_t}\{\tilde{a}_{t+1}\} = \mathrm{Var_t}\{a_{t+1}\} + [\gamma^z]^{-1} = \gamma_{t+1}^{-1} + [\gamma^z]^{-1}$. As a result, beliefs about future output y_{t+1} can be constructed from productivity beliefs θ_{t+1} and γ_{t+1}. Using the law of motion for beliefs about a, as given by Eq. (20) and (22), we express the linear approximation of beliefs about observed productivity, $\tilde{a}_{t+1}$, as follows

$$\mathrm{E_t}\{\tilde{a}_{t+1}\} = \mathrm{E_t}\{a_{t+1}\} = (1-\rho_a)\bar{a} + \rho_a[vs_t + (1-v)z_t], \tag{28}$$

$$\mathrm{Var_t}\{\tilde{a}_{t+1}\} = \gamma_{t+1}^{-1} + [\gamma^z]^{-1} = \sigma_a^2 + [\gamma^z]^{-1} + \frac{\rho_a^2}{\gamma}v(1+v\bar{y}) - \frac{\rho_a^2}{\gamma}v^2 \cdot y_t, \tag{29}$$

where $v \equiv \frac{\gamma}{\gamma+\gamma^z}$ is the steady-state weight given to signal s_t when forming beliefs.[20] Note that in Eq. (29), uncertainty about future productivity is decreasing in output, reflecting our assumption of procyclical precision of the signal. Intuitively, for $\gamma > 0$, a recession makes the signal s_t received by agents less informative (γ_t^s decreases) which corresponds to a raise in uncertainty, i.e. the variance of expected output increases. In response to higher uncertainty, households adopt a stronger precautionary-saving behavior (see Eq. (26)), which in turn reduces output and gives rise to the feedback effect of endogenous uncertainty.

We substitute the linear beliefs from Eq. (28) and (29) into Eq. (27), and then plug the result into the IS curve, Eq. (26). Expressing the result in terms of deviations from the steady state

[20]Due to our assumption of no prior information being available ($\gamma_t = 0$), beliefs are constructed based on two signals, s_t and z_t, with weights v and $1-v$, respectively.

gives the following expression for output

$$\hat{y}_t = f \cdot \left[-\frac{1}{\eta}\hat{r}_t + \frac{1+\omega}{\omega+\eta}\rho_a[v\hat{s}_t + (1-v)\hat{z}_t] + (1-\rho_g \cdot \frac{\omega}{\omega+\eta})\hat{g}_t \right], \tag{30}$$

where the parameter f is defined as

$$f \equiv \left[1 - \frac{1}{2} \cdot \frac{\rho_a^2(1+\omega)^2(1+\eta)}{(\omega+\eta)^2} \cdot \frac{\gamma}{(\gamma^z+\gamma)^2} \right]^{-1}. \tag{31}$$

Parameter $f \geq 1$ captures the intensity of amplification generated by our endogenous uncertainty channel.[21] When $f = 1$, the endogenous uncertainty channel is inoperative, resulting in no amplification. Conversely, when $f > 1$, the endogenous uncertainty channel amplifies the impact of shocks on output. We now briefly examine the conditions on the information flow under which the endogenous uncertainty channel is inactive. First, consider the case when signal s_t is highly precise, such that $\gamma \to \infty$. This scenario effectively represents a full information case, as agents learn about the true state of the economy with very high precision. Then, $f \to 1$, and the endogenous uncertainty channel becomes inoperative due to the absence of informational imperfections that would otherwise activate it. Said differently, the marginal information provided by economic activity is irrelevant as the signal received by agents is already perfect. Second, consider the case where the signal s_t is highly imprecise, with $\gamma \to 0$. In this scenario, we again have $f \to 1$, and the endogenous uncertainty channel becomes inoperative because the precision of information is insensitive to changes in output. Hence, the endogenous uncertainty channel emerges only for intermediate values where output provides relevant information to improve the precision of the signal, i.e. parameter γ is neither too high nor too low.[22]

4.2 Aggregate demand effect of productivity shock

We now turn to shed light on the precautionary-saving feedback effect induced by the endogenous uncertainty channel. To this end, we examine the economy's response to a productivity shock – an innovation in the level of the persistent productivity component. We demonstrate that the New-Keynesian result – that a negative productivity shock leads to overheated demand (Galí, 2008) – can be easily undermined and potentially overturned. In this latter case, the rise in aggregate uncertainty stimulates precautionary savings, which weakens aggregate demand, causing a negative output gap.

Following the New-Keynesian tradition, we use the output gap as our preferred measure of

[21]Note that $f \geq 1$ as long as $f > 0$. Values of $f \leq 0$ are economically implausible since amplification would be so strong that it would reverse the effect of shocks on aggregate output.

[22]Lorenzoni (2009) conducts a similar analysis in the context of the non-monotonic relationship between noise shock effects and signal precision.

aggregate demand. Using Eq. (24), we first define the flexible-price output as $y_t^f = \left(\frac{1+\omega}{\omega+\eta}\right)\tilde{a}_t + \left(\frac{\eta}{\omega+\eta}\right)g_t$, which is the output that would prevail in a flexible price economy. The output gap is then defined as the deviation of actual output from its flexible price counterpart, expressed as

$$\tilde{y}_t = y_t - y_t^f = \left(\frac{1}{\omega+\eta}\right) mc_t. \tag{32}$$

Note that in this economy, consumer price inflation also serves as a measure of aggregate demand, as it is fully determined by the output gap. To illustrate this, we log-linearize the Phillips curve (8), which, under our timing assumption about price stickiness yields

$$\pi_t = \frac{\epsilon}{\Phi} \cdot mc_t = \frac{\epsilon(\omega+\eta)}{\Phi}\tilde{y}_t. \tag{33}$$

We assume that monetary policy controls the real interest rate and targets the output gap according to the rule $\hat{r}_t = \phi\tilde{y}_t$. This is equivalent to inflation targeting, as inflation and the output gap are linked through the Phillips curve, as given by Eq. (33). The corresponding monetary rule for inflation targeting is $\hat{r}_t = \phi'\pi_t$, where $\phi' = \phi\frac{\Phi}{\epsilon(\omega+\eta)}$.

From Eq. (30), the output gap satisfies

$$\left[1 + \frac{\phi}{\eta} \cdot f\right] \cdot \tilde{y}_t = \frac{1+\omega}{\omega+\eta} \cdot \left[-\hat{\tilde{a}}_t + \rho_a f(v\hat{s}_t + (1-v)\hat{z}_t)\right] + f \cdot \left(1 - \frac{\omega\rho_g}{\omega+\eta} - \frac{\eta}{\omega+\eta}\right)\hat{g}_t. \tag{34}$$

To focus on the effect of productivity shocks, let us abstract from government spending by setting $\hat{g}_t = 0$, and from noise shocks with $\epsilon_t^z = 0$ and $\epsilon_t^s = 0$. Additionally, we also normalize log-productivity such that $\bar{a} = 0$. Under these assumptions, we have $\hat{z}_t = \hat{s}_t = \hat{\tilde{a}}_t = a_t$, and the link between productivity and the output gap is given by

$$\tilde{y}_t = -\psi(1-\rho_a) \cdot a_t + \psi\rho_a(f-1) \cdot a_t \tag{35}$$

where $\psi = \left[1 + \frac{\phi}{\eta} \cdot f\right]^{-1}\left[\frac{1+\omega}{\omega+\eta}\right]$. Eq. (35) gives the response of the output gap, $\tilde{y}_t$, to productivity shocks, a_t, decomposed through two effects. The first term on the right-hand side captures the standard New-Keynesian effect. As long as $\rho_a < 1$, the output gap, $\tilde{y}_t$, increases in response to a negative productivity shock because the actual output, y_t, decreases by less than its flexible-price counterpart, y_t^f. This outcome arises because pricing frictions prevent firms from raising prices to the level needed to achieve the flexible-price allocation following a negative productivity shock. As a result, the inefficiently low prices lead to overheated aggregate demand, reflected in a positive output gap. The second term in Eq. (35) arises when $f \neq 1$ and it captures the endogenous uncertainty channel in the transmission of productivity shocks. In particular, activating the endogenous uncertainty channel with $f > 1$ dampens the New-Keynesian effect, i.e. the increase in the output gap following a negative technology shock. Furthermore, if the endogenous uncertainty channel is sufficiently strong, such that $f > \rho_a^{-1}$, the output gap *decreases* in response to a negative productivity shock, revering the textbook New-Keynesian model result. In this case, the negative productivity shock becomes a "Keynesian supply shock", leading to a demand-driven recession.

To go deeper in our analysis, we compute the response of actual output and inflation to the productivity shock

$$y_t = y_t^f + \tilde{y}_t = f \cdot \psi \cdot \left[\frac{\phi}{\eta} + \rho_a\right] a_t, \tag{36}$$

$$\pi_t = \frac{\epsilon(\omega+\eta)}{\Phi} \cdot \tilde{y}_t = \frac{\epsilon\rho_a(\omega+\eta)}{\Phi}\left[f - \frac{1}{\rho_a}\right] a_t. \tag{37}$$

Eq. (36) makes clear that the endogenous uncertainty channel ($f > 1$) amplifies the output response to a productivity shock. On the opposite, Eq. (37) shows that the sign of the inflation response can be either positive or negative, depending on the intensity of the endogenous uncertainty channel. Specifically, when the channel is weak $f < \rho_a^{-1}$, a negative productivity shock results in a supply-like recession (output declines and prices rise). Conversely, when the channel is strong $f > \rho_a^{-1}$, a negative productivity shock leads to a demand-like recession (both output and prices go down).

In Figure 2, we graphically illustrate the adjustments of the economy following a negative productivity shock in the presence of endogenous uncertainty. The upper panel depicts the Aggregate Demand (AD) and Aggregate Supply (AS) curves in the output-inflation space. The AD curve is constructed by substituting the monetary rule $r_t = \bar{r} + \phi'\pi_t$ into the IS curve, Eq. (26). Similarly, the AS curve is obtained by substituting the output gap expression $\tilde{y}_t = y_t - y_t^f$ into the Phillips curve, Eq. (33).[23] The lower panel illustrates the law of motion of uncertainty alongside the IS curve in the output-uncertainty space. The "beliefs" line corresponds to Eq. (29). The IS line is derived by substituting the output beliefs from Eq. (27) into the IS curve specified by Eq. (26).

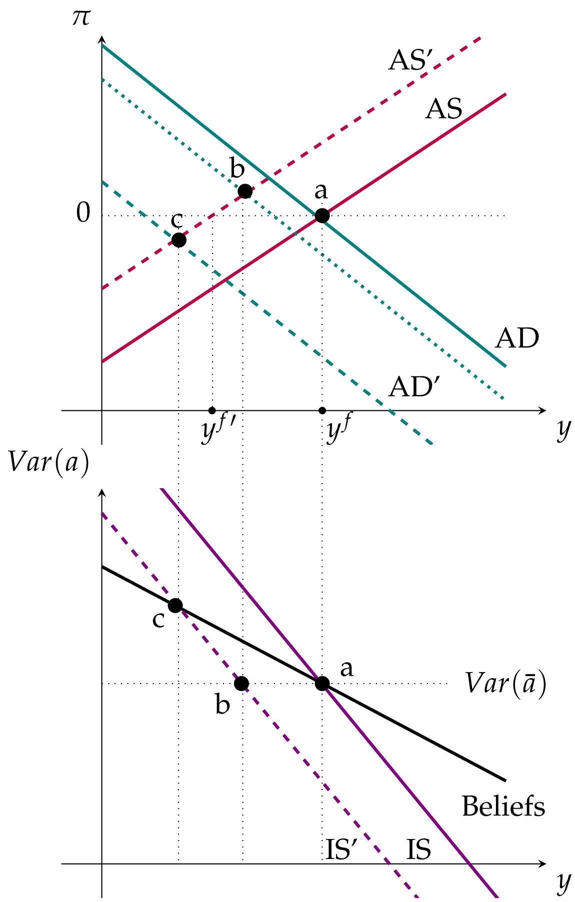
Let us begin by considering the economy initially at equilibrium point (a) with zero output gap ($y_t = y_t^f$). Let us assume a negative productivity shock that generates a drop in a_t reducing the flexible-price output from y_t^f to $y_t^{f\prime}$. This shifts the AS curve leftward from AS to AS′

[23]For the sake of clarity, we report here the AS and AD curve in their analytical form, such that

$$y_t = \eta^{-1}(\phi'\pi_t + \bar{r} - \rho) + \mathrm{E_t}\{y_{t+1}\} - \frac{1}{2}(1+\eta)\mathrm{Var_t}\{y_{t+1}\},$$

$$\pi_t = \frac{\epsilon(\omega+\eta)}{\Phi}(y_t - y_t^f), \text{ where } y_t^f = \left(\frac{1+\omega}{\omega+\eta}\right)(a_t + f_t).$$

Figure 2: Adjustments after negative productivity shock: illustration



indicating that, at each level of inflation, the economy is now capable of producing a lower level of output since the unit cost of production has gone up. All things equal, the intersection between AS' and AD illustrates the under-reaction of output in response to a supply shock, due to the presence of nominal rigidities (that makes the AS curve non-vertical). Since the level of output is smaller than $y_t^{f\prime}$, the output gap *increases* after a negative productivity shock. This all-things-equal supply-driven effect is combined with a typical demand-driven effect that arises when we take into account the expected income term, $\mathrm{E_t}\{y_{t+1}\}$ appearing in the IS curve. The drop in $\mathrm{E_t}\{y_{t+1}\}$ implies that output is less demanded, and thus the AD curve shifts downward. Altogether, these two effects drive us from point (a) to point (b), which represents a counterfactual equilibrium that would arise in the absence of the endogenous uncertainty channel. To see this, let's have a look at the lower panel of Figure 2, which shows that points (a) and (b) simply illustrate the leftward shift in the IS curve for a given level of uncertainty ($Var(a)$ is constant). In this constant-uncertainty point (b), output is lower but inflation is higher compared to point (a), indicating a supply-driven recession with a positive output gap. However, constant uncertainty does not hold in our economy, as the beliefs line is not horizontal but downward sloping. This captures the fact that lower output leads to higher uncertainty. Therefore, the lower panel of Figure 2 shows that the endogenous uncertainty channel drives the equilibrium point from (b) to point (c), which corresponds to the intersection between IS' and the downward-sloping beliefs line. Given the higher uncertainty, households respond with precautionary saving, which shifts the AD curve further downward, moving the economy to a new equilibrium at point (c), with even lower output and deflation. In this context, the endogenous uncertainty channel reverses the conventional New-Keynesian result, generating a demand-like recession in response to a negative productivity shock, as the shift from equilibrium (a) to equilibrium (c) leads to a decline in both output and price level. Notice that the precautionary saving is strong enough to generate an output smaller than its flexible-price counterpart, y'_f, implying a negative output gap.

To conclude this analytical part, let us briefly examine how the endogenous uncertainty channel affects the dynamics of hours worked. Equalizing labor supply and labor demand equations (Eq.(4) and (9), resp.), imposing the simplifying assumptions and applying a log-linear approximation gives us the following equilibrium expression for the labor market

$$l_t = \tilde{y}_t - \frac{\eta - 1}{\omega + \eta} a_t. \tag{38}$$

The equilibrium level of hours worked is negatively related to productivity when risk aversion $\eta \geq 1$, and they depend positively on the output gap. Provided uncertainty is constant, both the first and second terms in Eq. (38) go up in response to a negative productivity shock, implying an increase in hours, consistent with the baseline New-Keynesian result (Galí, 2008) since nominal rigidities imply that current output decreases by less than its flexible-part counterpart. On the labor market, a reduction in productivity generates a recession that reduces labor demand. However, this effect is compensated by the negative wealth effect on labor supply that drives consumption down and hours worked supply up. When the endogenous uncertainty channel is present, we have seen that the precautionary-saving effect is strong enough to reverse the sign of the output gap response to a productivity shock. Eq. (38) then shows that the response of labor to the negative productivity shock becomes ambiguous. On the one hand, one might expect that the precautionary-saving behavior that generates a magnified decrease in consumption leads to a stronger rise in labor supply. However, since endogenous uncertainty also amplifies the recession, the shift in the labor demand curve might dominate and generating a *reduction* in hours after a negative productivity shock.

4.3 Crowding out/in of private consumption

We also examine the role of the endogenous uncertainty channel in propagating the effects of a government spending shock on private consumption. For this exercise, we assume that $\hat{z}_t = \hat{s}_t = \hat{\tilde{a}}_t = 0$ and that $g_t \neq 0$, while monetary policy continues to respond to the output gap. The link between private consumption and government spending is given by

$$c_t = -\tilde{\psi}\left[\rho_g + \frac{\phi}{\eta}\right] \cdot g_t + \tilde{\psi}(f-1)\left[1 + \frac{\eta}{\omega} - \rho_g - \frac{\phi}{\eta}\right] \cdot g_t, \tag{39}$$

where $\tilde{\psi} = \frac{\omega}{1+\omega}\psi$. In our economy, government spending affects equilibrium consumption through two channels: the standard wealth effect and an additional channel arising from the endogenous impact of government spending on uncertainty.

The first term in Eq. (39) represents the standard New-Keynesian crowding-out effect. An increase in government spending, financed by a lump-sum tax, results in a reduction in private wealth. This triggers the negative wealth effect: since consumption and leisure are normal goods, a lower expected lifetime income reduces the demand for them. Consequently, in equilibrium, an expansion in government spending leads to a decline in private consumption, establishing a negative relationship between the two.

The second term represents the *crowding-in/out* effect arising from the endogenous uncertainty channel ($f > 1$). The rise in government spending boosts output, thereby reducing uncertainty. In response, households lower their demand for precautionary savings, leading to an increase in consumption. It is important to note that the crowding-in through the endogenous uncertainty channel is possible when $\phi/\eta + \rho_g < 1 + \eta/\omega$, which implies that government spending is not persistent (ρ_g small) or there is a weak monetary policy reaction (ϕ small).[24] Intuitively, a non-persistent positive public spending shock generates a short-lived negative wealth effect that dampens the traditional crowding-out in consumption. Similarly, a small reaction of the nominal interest rate to inflationary pressures – because the central bank is not active – also dampens the crowding-out in consumption. In both cases, the endogenous uncertainty channel dominates the traditional transmission channels of public spending shocks and generates a crowding-in in consumption.

[24] The importance of government spending persistence and the strength of the monetary policy response in the transmission of government spending shock was emphasized by Leeper et al. (2017). Here, we demonstrate that these characteristics are also crucial for the transmission, particularly through our endogenous uncertainty channel.

5 Numerical Illustration

Now, we proceed to the numerical evaluation of our endogenous uncertainty channel within the full-fledged non-linear New-Keynesian model laid out in Section 3.[25] To this end, we first calibrate the model on U.S. data and then we assess the role of endogenous uncertainty on the transmission channels of productivity and public spending shocks.

5.1 Parametrization

We calibrate the model at a quarterly frequency. There are two groups of parameters to be calibrated. The first group consists of standard New-Keynesian model parameters, which we calibrate to the conventional values in the literature and data. The second group is specific to our noisy information framework and governs the evolution of beliefs, particularly the precision of signals, denoted by γ^z and γ. We calibrate these parameters to match the moments of the ergodic distribution of aggregate forecast errors, as constructed in Section 2.2.

Table 2 reports the parameter values that are standard in New-Keynesian models and are calibrated using external sources or steady-state moments. The subjective discount rate is set to $\beta = 0.99$, corresponding to an annual interest rate of approximately 4% at the steady state. As is typical in the business cycle literature, the relative risk aversion is set to $\eta = 3$, and we assume a unitary Frisch elasticity of labor supply ($\omega = 1$). The elasticity of substitution between varieties is $\epsilon = 6$, implying a steady-state markup of 20% (in the absence of a correcting subsidy). The price adjustment cost is set $\Phi = 50$, which would correspond to an average price duration of roughly four quarters in a linearized model with Calvo staggering price setting. We assume that the interest rate elasticity to inflation in the Taylor rule is $\phi_\pi = 1.5$, while the elasticity to output is $\phi_y = 0.01$. We set returns-to-scale parameter $\alpha = 0.3$. The government spending-to-output ratio is set at 18%. Finally, following Fajgelbaum et al. (2017), the properties of the autoregressive process for the productivity shock are specified as $\sigma_a = 0.028$ and $\rho_a = 0.964$.

Next, we turn to the two parameters governing the evolution of beliefs: the precision of the signal z_t (γ^z) and the sensitivity of the signal s_t precision to output (γ), as these jointly determine the quantitative strength of the endogenous uncertainty channel. To calibrate these parameters we match the 5^{th} and the 95^{th} percentiles from the aggregate income growth forecast error series, $e^f_{t,t-12}$ constructed in Section 2.2. The 5^{th} percentile corresponds to periods of recession and slow recovery when perceived income growth is low and uncertainty is high due to low economic activity. The 95^{th} percentile corresponds to the times of expansion when perceived income growth is high and uncertainty is low.[26] To match these moments, we construct a similar object in the

[25] In line with the textbook New-Keynesian model of Galí (2008), we now assume that nominal bonds are the only saving vehicle, meaning that we consider the second expression in Eq. (3).

[26] In Appendix A, we plot the empirical distribution of forecast errors. Since forecast errors are more persistent in recessions (see our Section 2.2), this distribution is skewed to the left.

Table 2: Standard parameters

Parameter	Description	Value	Source/Target
β	Discount factor	0.99	Annual interest rate (4%)
η	Degree of risk aversion	3	Standard Value
ω^{-1}	Frisch elasticity of labor supply	1	Standard Value
α	Returns to scale	0.3	Standard Value
ϵ	Elast. of substitution btw goods	6	Markup (20%)
Φ	Price adjustment cost parameter	50	Price adjustment of one year
ϕ_π	Taylor rule parameter wrt inflation	1.5	Standard Value
ϕ_y	Taylor rule parameter wrt output gap	0.01	Standard Value
g/y	Government expenditure to output ratio	0.18	BEA (18%)
ρ_a	TFP shock: persistence	0.964	Fajgelbaum et al. (2017) estimate
σ_a	TFP level shock: s.d	0.028	Fajgelbaum et al. (2017) estimate

model – the household hourly income growth forecast error, $(\mathrm{E_t}\{\Delta \mathrm{y_t} - \Delta y_t)$. We estimate the parameters γ^z and γ by minimizing the sum of the squared distances between the empirical moments and the simulated model moments. Specifically, for each set of parameters $\{\gamma^z, \gamma\}$, we run one simulations matching the length of the empirical forecast error series. For each simulation, we compute the 5^{th} and 95^{th} percentiles of the distribution of the simulated forecast errors. Then we compute the average of these two quantities across simulations. Table 3 provides the empirical and model moments and the estimated parameter values.[27]

Table 3: Belief-related parameters and matched moments

Panel A: Parameters Value		
Parameter	**Description**	**Value**
γ	Sensitivity of signal s_t precision wrt output	130.0
γ^z	Precision of signal z_t	4.0

Panel B: Matched Moments		
Forecast error moment	**Data, %**	**Model, %**
5^{th} percentile	-6.43	-6.33
95^{th} percentile	5.33	5.42

[27] The moments of the empirical distribution are not perfectly matched because empirical forecast error may account for imperfect information about sources other than productivity, absent in our model.

5.2 IRFs Analysis

We now assess the role of the endogenous uncertainty channel in the propagation of aggregate shocks. To do this, we solve the model from the point of view of the representative household having imperfect information about productivity, resorting to a third-order perturbation technique setting all fundamental shocks to zero. Then, we build conditional simulations when the economy is hit either a negative productivity shock, corresponding to a negative supply shock. In addition, we extend our analysis by assessing the effects of a positive government spending shock. We refer to these simulations as the baseline model simulations. For each exercise, we also construct counterfactual IRFs by shutting down the endogenous uncertainty channel. In this counterfactual, the information is imperfect as in the baseline simulation; however, the level of uncertainty about productivity remains constant at its long-run value, which is taken from the baseline model.[28] We now proceed to discuss the results of these two simulations in turn.

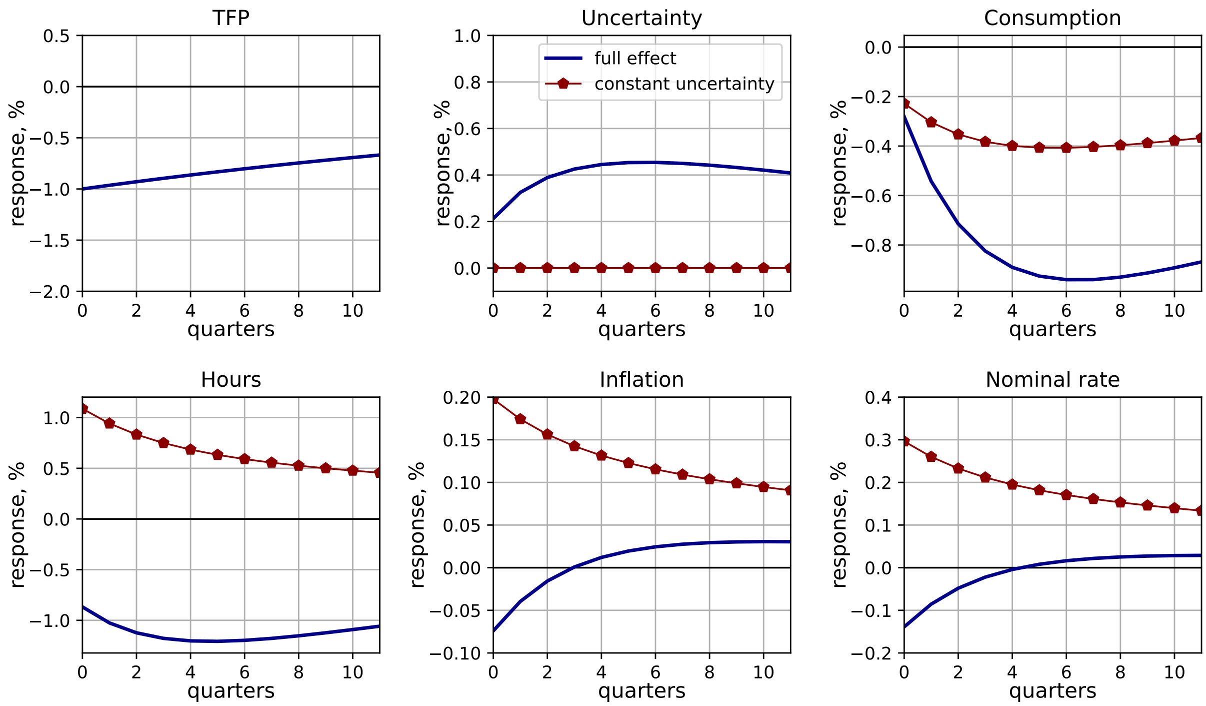
5.2.1 Productivity Shock

Figure 3 reports the IRFs to a negative productivity shock when the endogenous uncertainty channel is active (solid lines) and when it is shut down (dotted lines).

In the scenario with constant uncertainty, the effect of the negative supply shock aligns well with a standard New-Keynesian result. Inflation and output (proxied by consumption) co-move in the opposite direction, while inflationary pressures leads to a raise in the nominal interest rate. In addition, we observe that the number of hours worked increases, suggesting that the wealth effect on labor supply dominates on the negative labor demand effect, as explained by Section 4. Let now consider a model with time-varying uncertainty. In line with our analytical results presented in Section 4, the drop in consumption in response to a negative TFP shock is magnified by the presence of endogenous uncertainty. Intuitively, the recessionary effects of the shock reduces the precision of the signal – or raise uncertainty (i.e. $1/\gamma_t$) – which reinforces the precautionary motive of savers. As expected, this demand-driven recession is strong enough to make the negative supply shock *deflationary* (in line with our predictions from Figure (2)). Said differently, the drop in aggregate demand caused by heightened uncertainty over-weights the conventional effect of the productivity shock, leading to a demand-driven recession and turning the productivity shock into a Keynesian supply shock as output and inflation move in the same direction. When it comes to the response of hours worked, we find that they decrease, meaning that the recessionary effects leads to a strong decrease in labor demand.

[28]This counterfactual model closely resembles the representative agent noisy information New-Keynesian model of Lorenzoni (2009), with the caveat that while Lorenzoni (2009) analyzes linear behavior, our model captures nonlinear dynamics.

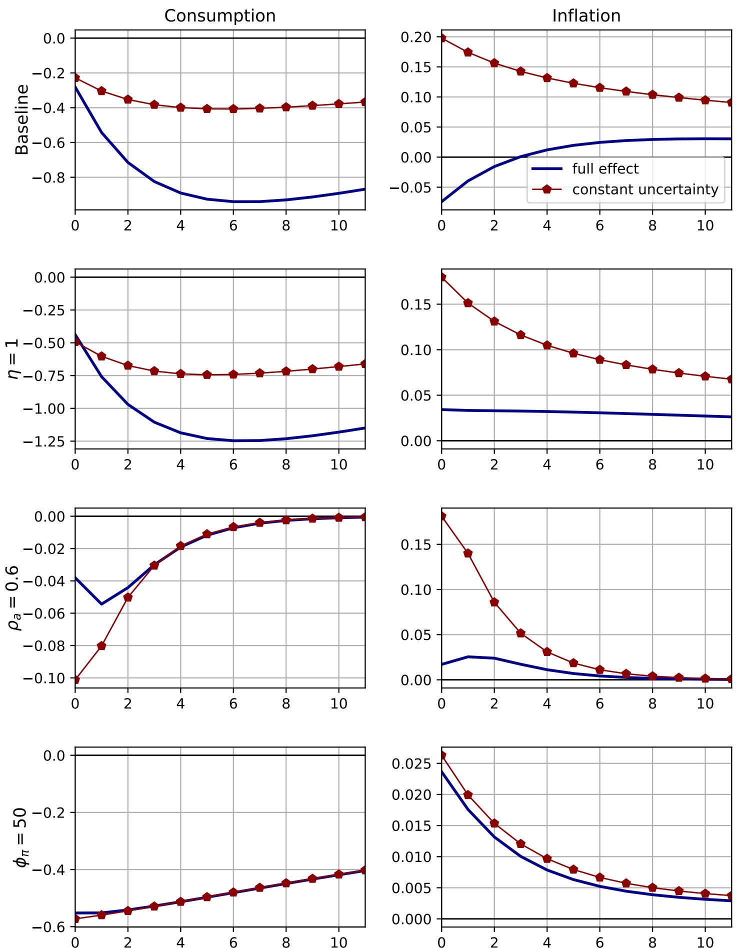
Figure 3: IRFs to a negative productivity shock



Note: The solid lines correspond to the response of the variables to a negative productivity shock in our baseline model with time-varying uncertainty. The dotted lines show the response in the counterfactual scenario where uncertainty is held constant. The response of uncertainty corresponds to the inverse of the response of γ_t. All IRFs are in deviation from their ergodic mean and multiplied by 100.

We go further in the analysis by investigating the conditions under which the endogenous uncertainty mechanism transforms a typical supply shock into a demand-like shock through a reinforced precautionary-saving channel. Figure 4 displays the responses of consumption and inflation to a negative productivity shock when the endogenous uncertainty channel is active (solid lines) or not (dashed lines) for different values of parameters. The first line corresponds to the IRFs in our baseline calibration. Let consider an unitary degree of risk aversion, $\eta = 1$, as display in the second line of Figure 4. By assuming a log-utility function, we mechanically reduce the strength of the precautionary-saving motive on consumption dynamics. On impact, consumption decreases by less than in the constant-uncertainty case. However, as the precautionary-saving channel is still active (see Equation (26)), consumption dynamics reacts negatively to the deterioration of information under endogenous uncertainty which ultimately generates a stronger recession over the long run. Notice that this demand-driven recession is not strong enough to reverse the sign of inflation response. The third line of Figure 4 compares the IRFs when the technology shock features a lower degree of persistence ($\rho_a = 0.6$). A short-lived productivity shock implies that the drop in the expected future income is not as strong as in the baseline,

Figure 4: IRFs to a negative productivity shock, parameters



Note: The solid lines correspond to the response of the variables to a negative productivity shock in our baseline model with time-varying uncertainty. The dotted lines show the response in the counterfactual scenario where uncertainty is held constant. All IRFs are in deviation from their ergodic mean and multiplied by 100.

which weakens the precautionary-saving behavior. Therefore, the amplification mechanism of endogenous uncertainty is reduced, as shown in Equation (31). Finally, we are interested in the strict inflation-targeting policy in the last line of Figure 4, by setting $\phi_\pi = 50$. Since Blanchard and Gali (2007), it is well accepted that a pure inflation-targeting rule allows the central bank to stabilize both inflation and the output gap in a New-Keynesian model, when the economy faces preference or technology shocks. Figure 4 confirms this result since the response of consumption is identical in both models, meaning that the effect of endogenous uncertainty on output dynamics disappears in this case. This finding suggests that an optimal monetary policy can offset the effects of the endogenous uncertainty channel.

5.2.2 Public Spending Shock

In Section 4, we emphasized that the endogenous uncertainty effect not only affects the transmission channels of supply shocks but also the demand shocks' one. Thus, we now turn to investigate the role of the endogenous uncertainty mechanism on the transmission channels of an expansionary public spending shock. Notice that we build this shock as a so-called "MIT" shock, meaning that the model is solved considering that the rise in government spending is received as a surprise by the agents but it follows a fully deterministic transition path. We assume that the spending-to-GDP ratio, g_t, follows an AR(1) process such that $g_t = (1-\rho_g)(g_t/\bar{g}) + \rho_g g_{t-1} + \sigma_g \epsilon_{g,t}$, where we calibrate $\sigma_g = 0.01$. In practice, we set $\epsilon_{g,t}$ equals to 1 in t and 0 afterward.

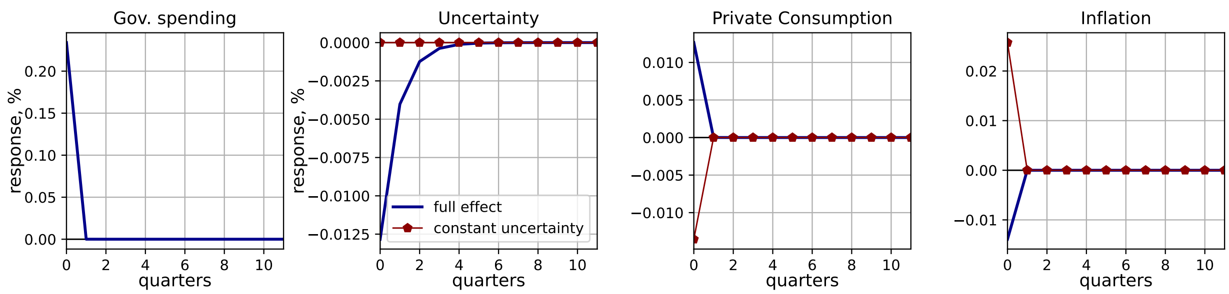
Panel (a) of 5 presents the IRFs with the baseline calibration and setting $\rho_g = 0.6$. As commonly found in the New-Keynesian literature, a positive public spending shock leads to crowding-out of private consumption. This result arises because increased public spending generates a negative wealth effect through higher taxes, prompting households to consume less and work more. However, in the presence of an endogenous response of uncertainty, the expansionary impact of the positive public spending shock reduces the level of uncertainty, thereby mitigating households' precautionary saving behavior, which, in turn, offsets the crowding-out effect on consumption. Notably, in the absence of an endogenous uncertainty channel, there is a positive aggregate demand effect, with both output and inflation increasing. In contrast, when the endogenous uncertainty channel is active, there is no overheated demand, and inflation goes down. The reason is that the reduction of uncertainty leads to an increase in flexible-price output, which is stronger than the actual increase in output (see Section 4). In other words, in the presence of endogenous uncertainty, government spending can stimulate the economy's potential production capacity through the information generated by economic activity.

As we demonstrated in the theoretical analysis, the role of the endogenous uncertainty channel in the transmission of a government spending shock depends on both the persistence of the shock, ρ_g and the strength of the monetary policy reaction function, ϕ_π. To explore these insights quantitatively, we conduct set of parametrization experiment. Panel (b) of Figure 5 displays the IRFs in which our economy is perturbed by a positive transitory ($\rho_g = 0$). It confirms the finding in our quantitative model – endogenous uncertainty channels have no mitigating effect on the response of private consumption to persistent shock. Both the standard and endogenous uncertainty channels operate in the same direction, transmitting the effects of the government spending shock on private consumption, consistent with our result in Section 4.

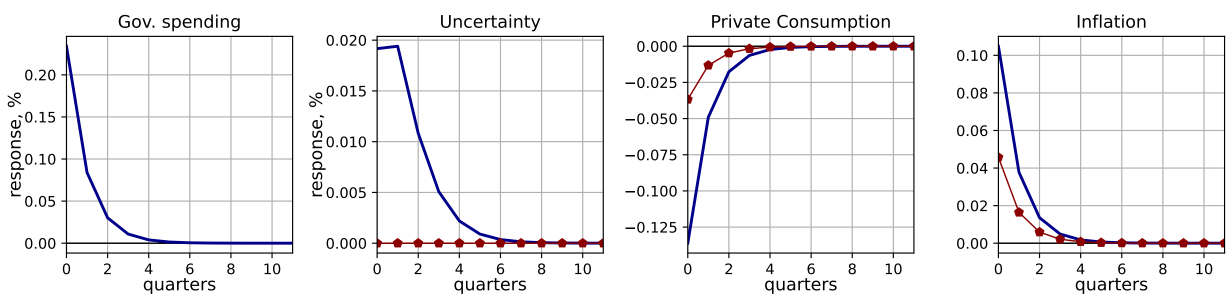
Finally, panel (c) of Figure 5 examine our baseline model-based IRFs under baseline calibration ($\rho_g = 0.7$) and strict inflation targeting rule ($\phi_\pi = 50$). As previously for the TFP shock analysis, the role of endogenous uncertainty channel is shut down by the strong reaction of the nominal interest rate.

Figure 5: Effect of positive government spending shock.

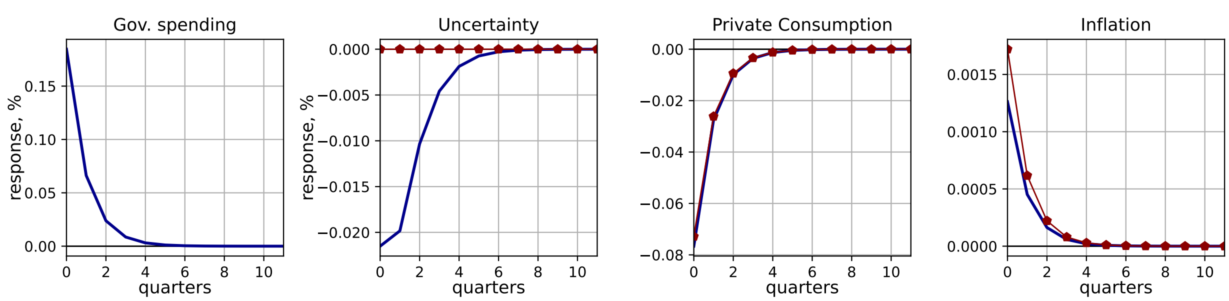
(a) Low persistence & baseline MP reaction



(b) Baseline persistence & MP reaction



(c) Baseline persistence & Strong MP reaction



Note: The solid lines correspond to the response of the variables to a positive public spending shock in our baseline model with time-varying uncertainty. The dotted lines show the response in the counterfactual scenario where uncertainty is held constant. Panel (a) corresponds to the baseline calibration ($\rho_g = 0.6$ and $\phi_\pi = 1.5$). In Panel (b), we set $\rho_g = 0$. In Panel (c), we set $\phi_\pi = 50$. The response of uncertainty corresponds to the inverse of the response of γ_t. All IRFs are in deviation from their ergodic mean and multiplied by 100.

6 Conclusion

This paper assesses empirically and theoretically the role of endogenous uncertainty – captured through a pro-cyclical precision of information received by consumers – on the transmission channel of supply shocks. We first show that accounting for the cyclicality of aggregate uncertainty in U.S. data, when a TFP shock hits the economy, it generates a significantly stronger recession and less inflationary pressure. This goes in favor of a demand-like supply shock. Using household-level data, we also stress that information frictions are stronger during recessions. With this evidence in hand, we build a noisy-information non-linear New-Keynesian model where the precision of the signal depends positively on the level of economic activity. We emphasize analytically how a "learning-by-doing" signal acts as a demand-amplification mechanism after a negative TFP shock: the implied recession increases consumer's misperception, which in turn gives more incentive to build precautionary saving and leads to a reduction in aggregate demand. This generates an increase in output gap and in inflation, what is commonly called a Keynesian supply shock. Resorting to a quantitative model, parametrized to replicate some empirical moments related to forecast errors, we confirm our result that endogenous uncertainty generates an aggregate demand amplification mechanism through the precautionary saving motive.

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Supply Shocks in the Fog
Appendix

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A Data Description

A.1 Data Sources

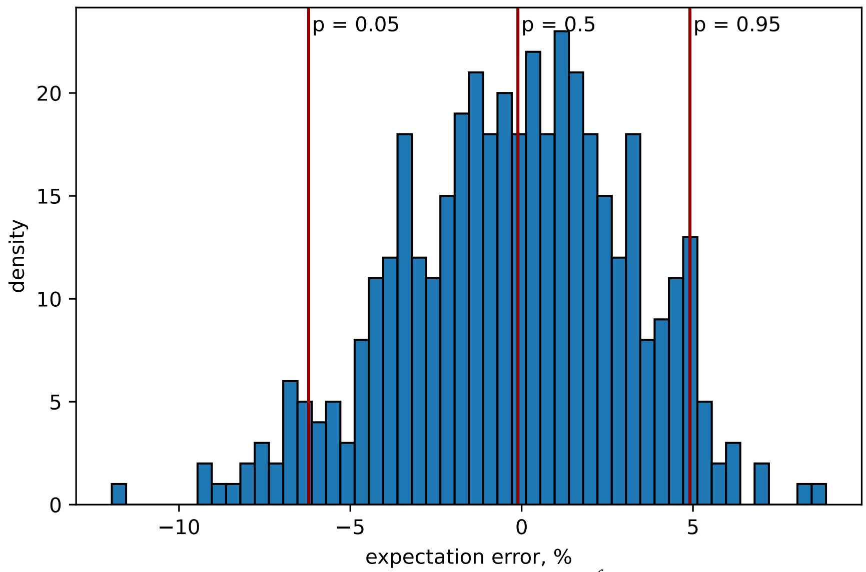
Table A.1. Data

(1): Real Gross Domestic Product	BEA	GDPC1
(2): Real Personal Consumption Expenditures	BEA	PCECC96
(3): Personal Consumption Expenditures, Durables	BEA	PCDG
(4): Real Gross Private Domestic Investment	BEA	GPDIC1
(5): Hours Worked for All Workers (Nonfarm)	BLS	HOANBS
(6): Chain-type Price Index, PCE	BEA	PCECTPI
(7): Compensation of Employees, Paid	BEA	A4102C1Q027SBEA
(8): Long-Term Government Bond Yields: 10-Year	OECD	IRLTLT01USQ156N
(9): Stock Price, SP500	Robert Shiller's website	Link
(10): TFP, utilization-adjusted	Fernald (2012)	Link
(11): Business Formation	Brand et al. (2019)	
(12): Consumer uncertainty	MSC	Link

Data sources: BEA: Bureau of Economic Analysis. BLS: Bureau of Labor Statistics. MSC: Michigan Survey of Consumers. OECD: The Organization for Economic Co-operation and Development.
Notes: All series are seasonally adjusted series and variables (1)-(6) and (9)-(12) expressed in log. The last column corresponds to the FRED database variable's name. **Real consumption of durables is (3)/(Deflator)**. Variable (9) is converted from a monthly to a quarterly frequency by taking the value from the last month of each quarter. Variable (11) is obtained by merging business formation data from the BEA' New Business Incorporations (for the period before 1995Q1) with establishment birth data from the BLS (starting from 1995Q1).

A.2 Michigan Survey of Consumers data

Figure 6: Forecast error distribution



Note: the figure displays the distribution of the average Forecast error, $e^f_{t,t-12}$, computed from the Michigan Survey of Consumers.

B Learning under imperfect information with procyclical precision

We derive the process the recursive process for aggregate forecast error in the Bayesian learning framework with imperfect information and procyclical precision. Let aggregate income follow the AR1 process:

$$inc_t = \rho \cdot inc_{t-1} + (1-\rho) \cdot \bar{inc} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

An individual household has income is:

$$inc^i_t = inc_t + v^i_t, \quad v^i_t \sim \mathcal{N}(0, (\gamma^v_t)^{-1})$$

v^i_t captures the individual component of income; γ^v_t is a precision parameter governing the dispersion of income distribution across individuals. The household does not observe inc_t directly and has a set of information Ω^i_t available at the beginning of period t. The corresponding prior belief about inc_t is

$$inc_t | \Omega^i_t \sim \mathcal{N}(\theta^i_t, (\gamma_t)^{-1})$$

where $E[inc_t|\Omega_t^i] = \theta_t^i$. During t household observes g_t^i and a noisy signal about inc_t

$$s_t^i = inc_t + u_t^i, \quad u_t^i \sim \mathcal{N}(0, (\gamma_t^u)^{-1})$$

Define expectation error as $e_{t-1}^i = \theta_t^i - inc_t^i$. Household updates information about inc_t so that: $E[inc_{t+1}|\Omega_{t+1}^i] = \rho E[inc_t|\Omega_{t+1}^i] + (1-\rho)\bar{inc}$ where the optimal combination of signals yields $E[inc_t|\Omega_{t+1}^i] = \frac{\gamma_t\theta_t^i + \gamma_t^v \cdot inc_t^i + \gamma_t^u s_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u}$ and $\gamma_{t+1} = \left[\frac{\rho^2}{\gamma_t + \gamma_t^v + \gamma_t^u} + \sigma^2\right]^{-1}$. The individual expectation error is then:

$$\begin{aligned} e_t^i = \theta_{t+1}^i - inc_{t+1}^i = \rho\frac{\gamma_t\theta_t^i + \gamma_t^v \cdot inc_t^i + \gamma_t^u s_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u} + E_t^i v_{t+1}^i - \rho \cdot inc_t - v_{t+1}^i - \varepsilon_{t+1} = \\ = \rho\frac{\gamma_t e_t^i + (\gamma_t + \gamma_t^v)v_t^i + \gamma_t^u u_t^i}{\gamma_t + \gamma_t^v + \gamma_t^u} + E_t^i v_{t+1}^i - v_{t+1}^i - \varepsilon_{t+1} \end{aligned}$$

Taking cross-sectional expectations (aggregating across individuals) and using that $E[v_t^i] = E[u_t^i] = E[E_{t-1}^i v_t^i] = 0$ because noise is idiosyncratic we obtain the average expectation error $e_t = E[e_t^i]$ as:

$$e_{t+1} = \rho\frac{\gamma_t}{\gamma_t + \tilde{\gamma}_t} \cdot e_t - \varepsilon_{t+1}$$

where $\tilde{\gamma}_t = \gamma_t^v + \gamma_t^u$ is the joint precision of two noisy signals. For a given prior precision γ_t, the persistence of forecast error is decreasing in signal precision $\tilde{\gamma}_t$. Finally, note that this aggregation relies on the fact that the precision of belief is the same across individuals, even though the mean belief is idiosyncratic.

C Alternative learning frameworks

As a first possibility, consider a standard "learning-by-doing" assumption that each unit of production generates information. A noisy productivity signal for j-th unit of good produced

$$s_t(j) = a_t + \epsilon_t^s(j), \;\; \epsilon_t^s(j) \sim \mathcal{N}(0, \; \gamma^{-1})$$

The total amount of goods produced is $\sum j = Y_t$. Then the average of these signals generated the overall noisy signal of precision equal the sum of precisions of the underlying signals:

$$s_t = \frac{1}{Y_t}\sum s_t(j) = a_t + \epsilon_t^s, \;\; \epsilon_t^s \sim \mathcal{N}(0, \; [\gamma \cdot Y_t]^{-1}) \tag{C.1}$$

which yields pro-cyclical precision of information flow from signal s_t.

The second possibility is to consider a combination of learning by doing and social learning, see Foster and Rosenzweig (1995). Let us assume that each worker i (out of total employment L_t)

gets a noisy signal about productivity for each unit j produced (learning by doing)

$$s_t(i,j) = a_t + \epsilon_t^s(i,j),\ \epsilon_t^s(i,j) \sim \mathcal{N}(0,\ \gamma^{-1}) \tag{C.2}$$

Each worker produces $y_t = \frac{Y_t}{L_t}$ goods, hence has an overall signal about productivity computed as the average of her learning by doing signals

$$s_t(i) = a_t + \epsilon_t^s(i),\ \epsilon_t^s(i) \sim \mathcal{N}(0,\ (\gamma \cdot y_t)^{-1}) \tag{C.3}$$

Finally, workers meet and exchange their information about productivity (social learning). The overall signal is the average of worker-specific signals and has precision $\gamma \cdot y_t \cdot L_t = \gamma_t \cdot Y_t$

$$s_t = a_t + \epsilon_t^s,\ \ \epsilon_t^s \sim \mathcal{N}(0,\ [\gamma \cdot Y_t]^{-1}) \tag{C.4}$$

D Risk-adjusted linear model

Market clearing. Reduced form price rigidity only in the present period: $MC_t \neq P_t$ but $MC_{t+j} = P_{t+j}$ for all $j > 0$. Combining (E.2), (E.3), and (E.6) and log-linearizing, we obtain

$$y_t = \left(\frac{1}{\omega+\eta}\right) mc_t + \left(\frac{1+\omega}{\omega+\eta}\right) \tilde{a}_t + \left(\frac{\eta}{\omega+\eta}\right) g_t \tag{D.1}$$

$$y_{t+j} = \left(\frac{1+\omega}{\omega+\eta}\right) \tilde{a}_{t+j} + \left(\frac{\eta}{\omega+\eta}\right) g_{t+j}, \quad j > 0 \tag{D.2}$$

IS equation. Assume that the household saves only in real bonds. We perform a risk-adjusted log-linearization of a corresponding Euler equation E.1. There are two stages to this linearization: first, derive the risk premium (in the spirit of Skinner (1988)) and then do a standard log-linearization. Our target is to derive the Euler equation in the form of a certainty-equivalent Euler equation adjusted for risk premium. We define certainty-equivalent consumption as consumption that households would choose if future income was certain and equal to its expected value. First, consider a nonlinear Euler equation:

$$u'(C_t) = \beta(1+r_t)E_t u'(C_{t+1})$$

Consider the point $C_{t+1}^e = E_t C_{t+1}$. To derive the risk-premium arising due to the consumption uncertainty (a la Skinner) we take the 2nd order Taylor expansion for RHS around C_{t+1}^e

$$u'(C_t) = \beta(1+r_t)E_t\left\{u'(C_{t+1}^e) + u''(C_{t+1}^e)\cdot(C_{t+1} - C_{t+1}^e) + \frac{1}{2}u'''(C_{t+1}^e)\cdot(C_{t+1} - C_{t+1}^e)^2\right\}$$

Taking expectations form the RHS:

$$u'(C_t) = \beta(1+r_t)\left(u'(C^e_{t+1}) + \frac{1}{2}u'''(C^e_{t+1}) \cdot E_t(C_{t+1} - C^e_{t+1})^2\right)$$

We factor out the certainty-equivalent part:

$$u'(C_t) = \beta(1+r_t)u'(C^e_{t+1})\left(1 + \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})} \cdot E_t(C_{t+1} - C^e_{t+1})^2\right)$$

Multiplying and dividing by $(C^e_{t+1})^2$ we obtain the expression in terms of the relative risk aversion:

$$u'(C_t) = \beta(1+r_t)u'(C^e_{t+1})\left(1 + \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})}(C^e_{t+1})^2 \cdot E_t\left(\frac{C_{t+1} - C^e_{t+1}}{C^e_{t+1}}\right)^2\right)$$

This equation can be rewritten as:

$$u'(C_t) = \beta(1+r_t)(1+\psi_t)u'(E_tC_{t+1}) \tag{D.3}$$

where $\psi_t = \frac{1}{2}\frac{u'''(C^e_{t+1})}{u'(C^e_{t+1})}(C^e_{t+1})^2 \cdot E_t\left(\frac{C_{t+1}-C^e_{t+1}}{C^e_{t+1}}\right)^2$ is time-varying risk premium arising from uncertainty about future consumption.

We have CRRA utility $u(C) = \frac{C^{1-\eta}}{1-\eta}$. From now on, for the Euler equation and all equations that follow (including the low of motion of uncertainty), we work with *linear* approximations. Taking the logs from Equation D.3 we get

$$-\eta log(C_t) = log\beta + log(1+r_t) + log(1+\psi_t) - \eta log(E_tc_{t+1}) \tag{D.4}$$

Next we denote $c_t = log(C_t)$. Using the fact that $log(1+x_t) \approx x_t$ and $logE_tX_{t+1} = E_tlog(X_t)$ to the first order, we get

$$c_t = -\frac{1}{\eta}\log(\beta) + E_tc_{t+1} - \frac{1}{\eta}r_t - \frac{1}{\eta}\psi_t$$

With CRRA utility we have

$$\psi_t = \frac{1}{2}\eta(1+\eta) \cdot E_t\left(\frac{C_{t+1} - C^e_{t+1}}{C^e_{t+1}}\right)^2$$

Noting that $\frac{C_{t+1}-C^e_{t+1}}{C^e_{t+1}} \approx c_{t+1} - Ec_{t+1}$ and $E_t(c_{t+1} - E_tc_{t+1})^2 = Var_tc_{t+1}$, and substituting for the log-linear resource constraint around the steady state with $\bar{G} = 0$ such that $y_t = c_t + g_t$ (where $g_t = \frac{G_t}{\bar{Y}}$) we obtain:

$$y_t - g_t = E_ty_{t+1} - \rho_g g_t - \frac{1}{\eta}(r_t - \rho) - \frac{1}{2}(1+\eta)Var_ty_{t+1} \tag{D.5}$$

where $\rho = -log(\beta)$.

Evolution of beliefs. We linearize beliefs (E.8), (E.9) around a stationary point $\gamma_t = \bar{\gamma}$, $\gamma^s_t = \bar{\gamma}^s$, $\theta_t = s_t = \tilde{a}_t = \bar{a}$. We obtain

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a\bar{a} + \rho_a\frac{\bar{\gamma}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(\theta_t - \bar{a}) + \rho_a\bar{a} + \rho_a\frac{\gamma^z}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(\tilde{a}_t - \bar{a}) + \rho_a\frac{\bar{\gamma}^s}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}(s_t - \bar{a}) +$$
$$+ (\frac{\bar{\theta}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} - \frac{\bar{\gamma}\bar{\theta}+\gamma^z\bar{\tilde{a}}+\bar{\gamma}^s\bar{s}}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2})(\gamma_t - \bar{\gamma}) + (\frac{\bar{s}}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} - \frac{\bar{\gamma}\bar{\theta}+\gamma^z\bar{\tilde{a}}+\bar{\gamma}^s\bar{s}}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2})(\gamma^s_t - \bar{\gamma}^s) =$$
$$= (1-\rho_a)\bar{a} + \rho_a\frac{\bar{\gamma}\theta_t + \gamma^z\tilde{a}_t + \bar{\gamma}^s s_t}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}$$

Now, let us denote the prior information and productivity observation as a joint signal $z_t = \frac{\bar{\gamma}}{\bar{\gamma}+\gamma^z}\theta_t + \frac{\gamma^z}{\bar{\gamma}+\gamma^z}\tilde{a}_t$ of precision $\bar{\gamma}+\gamma^z$. Also, let us denote $v = \frac{\bar{\gamma}^s}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s}$. Then, we can rewrite the next period belief as:

$$\theta_{t+1} = (1-\rho_a)\bar{a} + \rho_a[vs_t + (1-v)z_t] \tag{D.6}$$

Assuming that $\gamma_t = \bar{\gamma}$, using the fact that $\gamma^s_t = \gamma\frac{Y_t}{\bar{Y}}$, and the definition of v we obtain

$$\gamma^{-1}_{t+1} = \frac{\rho_a^2}{\bar{\gamma}+\gamma^z+\bar{\gamma}^s} + \sigma_a^2 - \frac{\rho_a^2}{(\bar{\gamma}+\gamma^z+\bar{\gamma}^s)^2}(\gamma_t - \bar{\gamma} + \gamma^s_t - \bar{\gamma}^s) =$$
$$= \rho_a^2(\frac{v}{\bar{\gamma}^s} - \frac{v^2}{\bar{\gamma}^s}\frac{\gamma^s_t - \bar{\gamma}^s}{\bar{\gamma}^s}) + \sigma_a^2 = \rho_a^2(\frac{v}{\bar{\gamma}^s} - \frac{v^2}{\bar{\gamma}^s}(y_t - \bar{y})) + \sigma_a^2$$

which equals

$$\gamma^{-1}_{t+1} = \frac{\rho_a^2}{\gamma}v(1 - v(y_t - \bar{y})) + \sigma_a^2 \tag{D.7}$$

Finally, we are interested in $E_t\tilde{a}_{t+1}$ and $Var\tilde{a}_{t+1}$, which are obtained from the updated beliefs about a_{t+1} as

$$E_t(\tilde{a}_{t+1}) = E_t a_{t+1} = (1-\rho_a)\bar{a} + \rho_a[vs_t + (1-v)z_t] \tag{D.8}$$

$$Var(\tilde{a}_{t+1}) = \gamma^{-1}_{t+1} + \sigma_f^2 = \sigma_a^2 + \sigma_f^2 + \frac{\rho_a^2}{\gamma}v(1+v\bar{y}) - \frac{\rho_a^2}{\gamma}v^2 \cdot y_t \tag{D.9}$$

Output gap response to productivity shock. Combining IS equation (D.5) with (D.2) and beliefs (D.8), (D.9) we obtain

$$\left[1 - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2}\cdot\frac{\rho_a^2}{\gamma}\cdot v^2\right]\cdot y_t = -log(\beta) - \frac{1}{\eta}\cdot r_t + (1-\rho_g+\rho_g\frac{\eta}{\omega+\eta})g_t +$$
$$+ \frac{1+\omega}{\omega+\eta}\cdot((1-\rho_a)\bar{a} + \rho_a[vs_t + (1-v)z_t]) - \frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2}(\sigma_a^2 + \sigma_f^2 + \frac{\rho_a^2}{\gamma}v(1+v\bar{y}))$$

Taking the log deviations from the steady state, we get

$$\left[1-\frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2}\cdot\frac{\rho_a^2}{\gamma}\cdot v^2\right]\cdot\hat{y}_t=-\frac{1}{\eta}\hat{r}_t+\frac{1+\omega}{\omega+\eta}\rho_a[v\hat{s}_t+(1-v)\hat{z}_t]+(1-\rho_g\cdot\frac{\omega}{\omega+\eta})\hat{g}_t$$

Let us denote $f=\left[1-\frac{1}{2}\frac{(1+\omega)^2(1+\eta)}{(\omega+\eta)^2}\cdot\frac{\rho_a^2}{\gamma}\cdot v^2\right]^{-1}$, which gives

$$\hat{y}_t=-\frac{f}{\eta}\hat{r}_t+f\frac{1+\omega}{\omega+\eta}\rho_a[v\hat{s}_t+(1-v)\hat{z}_t]+f(1-\rho_g\cdot\frac{\omega}{\omega+\eta})\hat{g}_t \tag{D.10}$$

From D.1 we have the natural output (in log-deviation from the steady state) equal $\hat{y}_t^n=\frac{1+\omega}{\omega+\eta}\hat{\hat{a}}_t+\frac{\eta}{\omega+\eta}\hat{g}_t$. Let the output gap be denoted as $\tilde{y}_t=\hat{y}_t-\hat{y}_t^n$. Let the monetary policy response be such that $r_t=\phi\cdot\tilde{y}_t$. Then, from (D.10) we can write

$$\left[1+\frac{\phi}{\eta}\cdot f\right]\cdot\tilde{y}_t=\frac{1+\omega}{\omega+\eta}\cdot\left[-\hat{\hat{a}}_t+\rho_a f(v\hat{s}_t+(1-v)\hat{z}_t)\right]+f(1-\rho_g\cdot\frac{\omega}{\omega+\eta}-\frac{\eta}{\omega+\eta})\hat{g}_t \tag{D.11}$$

Now, consider the change in productivity Δa_t. Change in productivity is reflected in the corresponding change in $\Delta\hat{\hat{a}}_t=\Delta a_t$, $\Delta a_t\hat{s}_t=\Delta a_t$, and $\Delta\hat{z}_t=\frac{1}{1+\bar{\gamma}\sigma_f^2}\Delta a_t$ (since z_t is the combination of prior belief and observation of the current productivity); to simplify further, we assume that $\bar{\gamma}=0$, that is, no prior information is available about the productivity. Then the response of the output gap to productivity shock is computed from

$$\frac{\omega+\eta}{1+\omega}\left[1+\frac{\phi}{\eta}\cdot f\right]\Delta\tilde{y}_t=[\rho_a f-1]\cdot\Delta a_t=-(1-\rho_a)\cdot\Delta a_t+\rho_a(f-1)\cdot\Delta a_t \tag{D.12}$$

It is clear that $f\geq 1$ always, as is the effect of endogenous uncertainty channel. Endogenous uncertainty channel is absent when: 1) no learning from economic activity $\gamma=0$, then we have $v=0$ and $f=1$, or 2) full information model $\gamma\to\infty$ (or $\sigma_f^2=0$), with $v=1$ and $f=1$.

Crowding in private consumption. Now consider an increase in government spending Δg_t. The response of private consumption is $\Delta c_t=\Delta y_t-\Delta g_t$. The response of output to government spending shock is given from D.10 as $\Delta y_t=-\frac{1}{\eta}f\Delta r_t+f(1-\rho_g\cdot\frac{\omega}{\omega+\eta})\Delta g_t$ and the response of interest rate is given from D.11 as $\Delta r_t=\phi\Delta\tilde{y}_t=\frac{\phi f(1-\rho_g\cdot\frac{\omega}{\omega+\eta}-\frac{\eta}{\omega+\eta})}{1+\frac{\phi}{\eta}f}$. Then, the consumption response to government spending shock is

$$\Delta c_t=\left[-1+\frac{(1-\rho_g\cdot\frac{\omega}{\omega+\eta}+\frac{\phi}{\omega+\eta})f}{1+\frac{\phi f}{\eta}}\right]\Delta g_t$$

The response of consumption depends on the persistence of government spending. Rearranging the terms we get

$$\left[1+\frac{\phi f}{\eta}\right]\Delta c_t = (f-1)(1-\rho_g\cdot\frac{\omega}{\omega+\eta}-\frac{\phi}{\eta}\frac{\omega}{\omega+\eta})\Delta g_t - (\rho_g\cdot\frac{\omega}{\omega+\eta}+\frac{\phi}{\eta}\frac{\omega}{\omega+\eta})\Delta g_t \tag{D.13}$$

The first term is the crowding in effect from the endogenous uncertainty channel ($f > 1$). The second effect is the grounding out effect from the traditional government spending channel. When government spending is persistent (large ρ_g), the standard crowding out effect is strong and the endogenous uncertainty crowding in is weak.

E Numerical model appendix

In this section, we summarize the equations used in Section 5 and the resolution method.

E.1 Model equations

Household:

$$1 = \beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\eta}\frac{R_t}{\pi_{t+1}}\right\}, \tag{E.1}$$

$$W_t = L_t^{\omega}C_t^{\eta}. \tag{E.2}$$

Firm:

$$MC_t = \frac{1}{1-\alpha}\frac{W_t}{\tilde{A}_t}L_t^{\alpha}, \tag{E.3}$$

$$Y_t = \tilde{A}_t L_t^{\alpha}. \tag{E.4}$$

Price-setting:

$$\epsilon\left(1-MC_t\right) = 1-\Phi\left(\pi_t-1\right)\pi_t+\Phi \mathrm{E}_t\left\{Q_{t,t+1}\left(\pi_{t+1}-1\right)\pi_{t+1}^2\frac{Y_{t+1}}{Y_t}\right\}, \tag{E.5}$$

where the discount factor is $Q_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\eta}\frac{1}{\pi_{t+1}}$.

Market clearing:

$$Y_t = C_t + G_t + \frac{\Phi}{2}\left(\pi_t-1\right)^2 Y_t. \tag{E.6}$$

Monetary policy:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}. \tag{E.7}$$

Evolution of beliefs about a_t:

$$\theta_{t+1} = (1-\rho_a)\bar{a}+\rho_a\frac{\gamma_t\theta_t+\gamma^z z_t+\gamma_t^s s_t}{\gamma_t+\gamma^z+\gamma_t^s}, \tag{E.8}$$

$$\gamma_{t+1}^{-1} = \frac{\rho_a^2}{\gamma_t + \gamma^z + \gamma_t^s} + \sigma_a^2. \tag{E.9}$$

E.2 Solution and simulation details

We solve the rational expectation model from the point of view of a representative household having imperfect information about productivity. That is, beliefs θ_t and γ_t are the observable state variables but not the persistent productivity component a_t. As households compute their expectations based on these beliefs, they perceive deviation of signals from these beliefs as realizations of expectation errors. Given the prior information Ω_t, observing signals z_t and s_t consists of the expected part θ_t and the innovation part. Signal z_t is

$$z_t = \theta_t + \tilde{e}_t^z,$$

where $\tilde{e}_t^z = (a_t - \theta_t) + \epsilon_t^z$ is innovation, consisting of expectation error $a_t - \theta_t$ and realization of temporary productivity component $f_t = \epsilon_t^z$. Similarly, the public signal s_t is

$$s_t = \theta_t + \tilde{e}_t^s$$

where $\tilde{e}_t^s = (a_t - \theta_t) + \epsilon_t^s$ is a sum of expectation error $(a_t - \theta_t)$ and a realization if signal noise ϵ_t^s. Hence, treating errors $\tilde{e}_t^z$ and $\tilde{e}_t^s$ as exogenous disturbances (which they are from the point of view of a household) account for the imperfect information of the household in the dynamic RE model.

Let e_t^a denote the expectation error between realized productivity component, a_t, and the expected one, $\mathrm{E}(a_t|\Omega_t)$, such that $e_t^a \equiv a_t - \theta_t$. Notice that expectation error is drawn from the distribution $e_t^a \sim \mathcal{N}(0, \gamma_t^{-1})$. We can therefore rewrite the observed signals in terms of the expected element θ_t and standard normal innovations scaled by the corresponding (time-varying) standard deviations

$$z_t = \theta_t + e_t^a + \epsilon_t^z = \theta_t + [\gamma_t]^{-1/2}\epsilon_t^e + [\gamma^z]^{-1/2}\epsilon_t^z \tag{E.10}$$

$$s_t = \theta_t + e_t^a + \epsilon_t^s = \theta_t + \gamma_t^{-1/2}\epsilon_t^e + [\gamma_t^s]^{-1/2}\epsilon_t^s \tag{E.11}$$

where ϵ_t^e, ϵ_t^z and ϵ_t^s are i.i.d. drawn from a standard normal distribution. We solve the model consisting of equations (E.1) - (E.9) and (E.10) - (E.11) using third-order perturbation method, allowing us to capture the precautionary saving channel. Simulating the model response to shocks amounts to recursive construction of expectation error e_t^a and computing the model path conditional on its realization.