Age-specific income risk and consumption over the life cycle and business cycle*

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Abstract

Age and risk play key roles in shaping consumption patterns. This paper documents the age-specific nature of business cycle fluctuations in income risk and examines how old-age-specific labor income risk influences consumption dynamics across age groups. We identify a series of old-age-specific labor income risk shocks in the U.S. and find that the consumption of middle-aged workers responds most strongly to these shocks. We then assess whether a standard heterogeneous agent life-cycle model can replicate our empirical findings. The model aligns with the empirical evidence as long as it accounts for the differences in individuals' sensitivity to risk implied by the MPC and prudence heterogeneity.

JEL classification numbers: E21, E32, E24, J11

Keywords: age-specific income risk, business cycle, precautionary savings, consumption dynamics

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1 Introduction

Income risk and age are pivotal determinants of household consumption and savings behavior, and understanding how these factors interact over the business cycle is essential for effective macroeconomic policy. Existing literature highlights two distinct dimensions of income risk variation: the life cycle and the business cycle. Research on age-specific income risk indicates that income risk evolves in a U-shaped manner over the life cycle (Karahan and Ozkan, 2013; Guvenen et al., 2021), while studies on the business cycle demonstrate that labor income risk experiences cyclical fluctuations, nearly doubling during recessions (Storesletten et al., 2004; Bayer et al., 2019). Yet, little is known about the age-specific nature of the business cycle fluctuations in income risk. This paper aims to fill this gap by investigating the possibility that people of different age groups face different magnitudes of cyclical fluctuations in income risk. Specifically, we document that older workers face larger business cycle fluctuations in income risk compared to younger workers. Then we show that consumption of the middle-aged workers is particularly responsive to fluctuations in the old-age-specific labor income risk, even though their own income risk is not directly affected by these fluctuations. We interpret these empirical findings within the framework of standard consumption theory, employing a heterogeneous-agent overlapping generations (OLG) model.

We begin by documenting the age-specific nature of business cycle fluctuations in idiosyncratic labor income risk in the U.S. Our empirical approach extends the work of Storesletten et al. (2004) and Bayer et al. (2019) by allowing business cycle fluctuations in income risk to be age-specific. First, we quantify the income risk faced by each age group over time at a quarterly frequency by fitting a semi-structural dynamic income model to a set of cross-sectional second moments of income distribution derived from the Survey of Income and Program Participation (SIPP). Then, to capture the age-specific nature of income risk fluctuations, we impose a structure on the risk variable such that the total income risk faced by each age group at any given time is the sum of two components: a uniform risk component and an old-age-specific risk component.

The old-age-specific risk component disproportionately affects the income risk of older workers (aged 45 and above), while the uniform risk component affects the income risk of all age groups uniformly. We estimate the contributions of these two components to the overall income risk faced by each age group over time. We find that older workers consistently experience significantly larger business cycle fluctuations in income risk compared to younger workers. Subsequently, we compute the innovations in risk associated with these two components, which we refer to as the old-age-specific income risk shock and the uniform income risk shock. While our uniform risk shock is conceptually similar to the shock previously identified by Bayer et al. (2019), the novelty of our paper lies in the identification of an additional old-age-specific risk shock.

We proceed by evaluating the impact of old-age-specific income risk shocks on consumption across different age groups and contrast it with the effect of uniform risk shocks. For this purpose, we construct a comprehensive measure of age-specific consumption for the working-age population from the Consumer Expenditure Survey (CEX) survey. We then use the local projection method to estimate how age-specific consumption responds to both old-age-specific and uniform risk shocks. In response to old-age-specific risk shocks, the consumption of younger workers (ages 25-34) does not exhibit a significant reaction, and the consumption of older workers (ages 45-55) responds very moderately. At the same time, middle-aged workers (ages 35-44) experience a relatively strong drop in consumption, despite their immediate income risk not being affected by old-age-specific risk shocks. We interpret these results to suggest that our old-age-specific risk shock includes a "news" element informing middle-aged workers about their future risk perspectives.

We then compare the consumption response to the old-age-specific risk shock with a corresponding response to the uniform risk shock. Unlike the old-age-specific shock, the uniform shock induces the largest drop in consumption among the youngest workers and a less pronounced response among the middle-aged and older workers. Thus, the key difference between the effect of the old-age-specific and uniform risk shocks is that the old-age-specific shock induces the largest decrease in consumption among middle-aged workers, whereas the uniform risk shock causes the largest drop in consumption among younger workers. When analyzing consumption at a more granular level using 5-year age groups instead of 10-year age groups, our empirical evidence indicates a U-shaped age profile of the consumption response to the old-age-specific risk shock. Specifically, workers aged 40-44 exhibit the most pronounced decrease in consumption, whereas the response of younger and older workers is notably weaker.

Next, we assess the ability of a standard life-cycle consumption model featuring the precautionary saving motive to account for the empirical U-shaped age profile of consumption response to the old-age-specific risk shock. For this purpose, we employ a Hugget-type OLG model which allows for heterogeneity across age groups and wealth levels. From the model, we derive an approximate analytical response of age-specific consumption to a generic labor income risk shock, encompassing both the old-age-specific and the uniform risk shocks as special cases. The analytical characterization enables us to identify how key household characteristics shape the consumption response to risk shocks.

Specifically, we show analytically that the sensitivity of age-specific consumption to labor income risk depends on the distributions of the marginal propensity to consume out of permanent labor income (labor income MPC) and the degree of prudence to labor income risk (labor income prudence). Note that our labor income MPC and labor income prudence measures pertain to the permanent *labor* income as opposed to the lifetime permanent income. In the presence of retirement, our labor income MPC and prudence differ from the lifetime permanent income MPC and prudence measures commonly found in the literature.[1] To compute realistic distributions of labor income MPC and prudence across age and wealth groups, we calibrate our model for the U.S., accounting for stationary age profiles of productivity and risk, as well as survival rates.

Our quantitative results indicate that standard consumption theory can produce a U-shaped age profile of consumption response to the old-age-specific income risk shock consistent with empirical evidence, but only when realistic heterogeneity in labor income MPC and prudence is accounted for. The reason is that in the calibrated model, labor income MPC and prudence decrease with both age and wealth, thereby reducing the relative sensitivity of older and wealthier workers to income risk. Consequently, middle-aged workers exhibit the strongest reaction to the old-age-specific income risk shock due to two factors: 1) higher exposure to the old-age-specific income risk compared to younger workers (due to the proximity to older age), and 2) higher sensitivity to risk compared to older workers. Finally, we investigate separately the contribution of age heterogeneity of labor income MPC and prudence as opposed to wealth heterogeneity in generating the U-shaped age profile consumption response. We find that age heterogeneity alone is not enough to reproduce the empirical evidence.

The paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents the empirical evidence on the age-specific nature of income risk and its effect on consumption across age groups. Section 4 provides a theoretical evaluation of the impact of the old-age-specific income risk shock. Section 5 concludes.

[1] The presence of retirement causes labor income MPC to decrease with age in the standard consumption model, in contrast to the lifetime permanent income MPC, which stays constant over the life-cycle. Thus, our MPC measure behaves as a blend of permanent and transitory income MPC, see Fagereng et al. (2021).

2 Related literature

Our paper relates to the literature dealing with the implications of time-varying idiosyncratic labor income risk. Two related studies by Storesletten et al. (2001, 2004) explore the cross-sectional variation of household income from the Panel Study on Income Dynamics. Both papers report that the income variance changes countercyclically and that it roughly doubles during recessions. Bayer et al. (2019) estimate shocks to the second moment of the labor income distribution and find that positive risk shocks accompany economic downturns, with important implications for the aggregate activity and household balance sheet. A common feature of this literature is that income risk fluctuations are uniform across age groups. In contrast, we focus on the age-specific component of income risk fluctuations over the business cycle.

Our paper also relates to studies documenting that income risk varies with age. Karahan and Ozkan (2013) reject the hypothesis that the conditional variance of the permanent income shocks has a flat life cycle profile. They partially confirm the previous findings of Meghir and Pistaferri (2004), with the effect of age being a marginally significant determinant of variance of income innovations. The work of Guvenen et al. (2021) documents the U-shaped pattern of the dispersion of earnings growth shocks across age groups for most earning groups. Our paper contributes to the literature on age-specific income risk by introducing time variation in the age profile of income risk. We document that the age profile of labor income risk fluctuates over the business cycle and examine the effect of these age-specific risk fluctuations on consumption. In this sense, we relate to the literature emphasizing age as one of the most important determinants of consumption dynamics Attanasio and Browning (1995).

In a broader sense, our paper relates to a stream of literature that forms modern consumption theory emphasizing the precautionary saving motive. This theory has a long tradition of reconciling dynamic economic models, which treat consumption as the solution to an intertemporal optimization program, with existing empirical findings regarding the behavior of aggregate consumption time series. The seminal paper by Hall (1978) incorporates rational expectations into household behavior. His solution implies the certainty equivalence, with consumption following a random walk. However, the certainty equivalent consumption model cannot explain numerous empirical puzzles known as "excess sensitivity" (Flavin, 1981), "excess smoothness", and "excess growth" puzzles (Deaton, 1986). Carroll (2001) argues that estimates of the first-order approximation of the Euler equation are likely to suffer from the omitted variable bias. Caballero (1990) shows that relaxing the assumption of certainty equivalence and allowing for precautionary savings can resolve all three puzzles. Carroll and Samwick (1997), Jappelli and Pistaferri (2000) document the evidence of precautionary saving in the household balance sheet, while Blundell et al. (2008) empirically reject the hypothesis of complete insurance. We follow this literature in studying the importance of precautionary motives for consumption behavior. In contrast to this classical consumption literature, we treat income risk as varying with time and age.

Finally, our paper is related to growing research on the aggregate consequences of time-varying uncertainty. Theoretical works in this stream of research show that variation in uncertainty can cause significant aggregate fluctuations through the real option effect (Bloom et al., 2007), precautionary saving (Fernández-Villaverde and Guerrón-Quintana, 2020), Oi-Abel-Hartman effect, etc.[2] The uncertainty shocks literature scrutinizes different facets of time-varying uncertainty. Bloom et al. (2018) shows that a shock to the second moment of total productivity generates a rapid drop in output and employment. Fernández-Villaverde et al. (2015), Born and Pfeifer (2014) show that policy uncertainty shocks have a quantitatively significant effect on output. Our paper falls into a complementary stream of literature that focuses on the properties and outcomes of idiosyncratic cyclical labor income risk rather than

[2] See Bloom (2014) for the list of the theoretical mechanisms.

investigating the implications of aggregate uncertainty.

3 Empirical evidence

This section empirically explores the age-specific nature of business cycle fluctuations in income risk. Our empirical approach extends the methodology of Storesletten et al. (2004) and Bayer et al. (2019) by allowing business cycle fluctuations in income risk to be age-specific. First, we document that older workers face more income risk fluctuations over the business cycle compared to younger workers. Then, we construct a set of old-age-specific income risk shocks, which we define as innovations to the difference in the income risk faced by older and younger workers. We then estimate the effect of these shocks on consumption across age groups and contrast it to the corresponding effect of uniform income risk shocks. The section now turns to the description of the data and the dynamic income model used in our empirical analysis. We then proceed with the estimation strategy and results.

3.1 Data description

We use two surveys of the U.S. population in our analysis: Survey of Income and Program Participation (SIPP) and Consumer Expenditure Surveys (CEX).

Survey of Income and Program Participation (SIPP). To measure the age-specific income risk, we use the individual-level labor income data from the Survey of Income and Program Participation (SIPP) conducted by the U.S. Census Bureau. SIPP is a set of monthly panels, containing the information on the individual members of each sample unit. Following SIPP guidelines, we identify a household as all sample unit members sharing the same address. We focus on households, rather than individuals, to account for within-household risk sharing. Within each household, we compute the number of children as the count of household members under the age of 18. The male and female heads are identified as the oldest man and woman in the household, respectively. We define the age of a household as the age of the oldest head. We keep only those households where the male and female heads report being married. We construct household income as a sum of labor earnings of the male and female heads. We retain only households with ten or fewer members. To focus on the working-age population (rather than retired workers), we limit our sample to households where the age of the heads ranges between 25 and 55 years. We remove households that worked less than one hour during a quarter (thus excluding households with prolonged unemployment periods). We keep only households with three monthly observations per quarter and aggregate observations to a quarterly frequency. Then, we impute household taxes and transfers using the TAXSIM network service provided by NBER[3] and construct after-tax household labor income. Finally, we retain observations only for the period spanning 1983Q4 to 2013Q2, as later periods were subject to a change in survey design.[4]

Consumer Expenditure Surveys (CEX). For the age-specific consumption measures, we use the consumption expenditure data from the Consumer Expenditure Surveys (CEX) conducted by the U.S. Bureau of Labor Statistics. Within the CEX survey, we use the FMLI files from the Interview Survey. These files contain quarterly expenditure summaries at the Consumption Unit level, encompassing total consumption including durable goods, housing, and other broad consumption items (a similar consumption measure commonly used to construct the aggregate consumption component of GDP). We exclude Consumption Units where the reference person is younger than 25 or older than 55. Then, we classify Consumption Units into three age groups

[3]For the TAXSIM model description see Feenberg and Coutts (1993). The TAXSIM NBER service is accessible via https://users.nber.org/∼taxsim/

[4]Staring from 2014, SIPP changed its survey design, resulting in lack of compatibility with previous dates (see SIPP 2014 User's Guidelines). Please also refer to Appendix A where we plot the number of observations in our cleaned dataset for each quarter – the number of suitable observations significantly drops starting from 2014.

(25-34, 35-44, and 45-55 years old) and calculate the weighted average consumption expenditure within each age group (using FMLI sample weights). This will be referred to as age-specific consumption. Additionally, we also repeat this procedure for a more detailed disaggregation into six age groups (25-29, 30-34, 35-39, 40-44, 45-49, 50-55).

3.2 Dynamic income model with age-specific risk

Now we turn to the description of our dynamic income model, which is an extended version of Storesletten et al. (2004); Bayer et al. (2019) model, featuring *age-specific* business cycle fluctuations in income risk.

3.2.1 Income process

The logarithm of income of a household i at time t is y_{it}. Household income consists of a deterministic element $f(X_{it})$, which depends on the aggregate and idiosyncratic characteristics X_{it}, and a stochastic element u_{it}

$$y_{it} = f(X_{it}) + u_{it} \tag{1}$$

The deterministic element includes observable features that predict household income levels. The stochastic element consists of an individual fixed effect μ_i, a transitory component τ_{it}, and a permanent component h_{it}

$$u_{it} = \mu_i + \tau_{it} + h_{it} \tag{2}$$

with the individual fixed effect being normally distributed across households $\mu_i \sim N(0, \sigma_\mu^2)$ and the transitory component following MA(1) process $\tau_{it} = \epsilon_{it}^\tau + \rho_\tau \epsilon_{it-1}^\tau$, $\epsilon_{it}^\tau \sim N(0, \sigma_\tau^2)$. The distribution of individual fixed effects is generally cohort-specific.

The permanent component h_{it} accumulates all the income shocks that have occurred to household i from birth until the present moment, with a discount rate of ρ_h for past shocks. Let g be the age of a household at time t. The permanent component of income is

$$h_{it} = \sum_{s=c}^{t} \rho_h^{t-s} \epsilon_{i,s,s-c}^h, \quad \epsilon_{i,t,g}^h \sim N(0, \sigma_{t,g}^2) \tag{3}$$

with $c = t - g$ is the birth date (cohort), s runs through all periods from birth date to the present time t, $t - s$ is the remoteness of period s from the present moment, $s - c$ is the age at time s.

Notably, the distribution of income shocks faced by a household of age g at time t depends not only on time but also on the household's age (as indicated by the subscript g in $\sigma_{t,g}^2$). This distinguishes our model from Bayer et al. (2019).

3.2.2 Age-specific income risk

From Equation (3), the cross-sectional variance of the permanent income component for age g at time t is a discounted sum of the permanent income shock variances, capturing the ex-ante income risk faced by the household. This sum can then be written in recursive form

$$\sigma_h^2(t, g) = \sum_{s=c}^{t} \rho_h^{2(t-s)} \sigma_{s,s-c}^2 = \rho_h^2 \cdot \sigma_h^2(t-1, g-1) + \sigma_{t,g}^2 \tag{4}$$

Let us assume that the contemporaneous risk $\sigma_{t,g}^2$ faced by a household of age g is a combination of three terms: a "common" risk σ_t^2, a "young" risk $\sigma_{y,t}^2$, and an "old" risk $\sigma_{o,t}^2$. The "young" risk component affects only households younger than $\hat{g}$ years, while the "old" risk component affects only households older than $\hat{g}$. That is,

$$\sigma_{t,g}^2 = \sigma_t^2 + (1 - I_{g>\hat{g}}) \cdot \sigma_{y,t}^2 + I_{g>\hat{g}} \cdot \sigma_{o,t}^2 \tag{5}$$

where $I_{g>\hat{g}}$ is an indicator, taking the value of 1 if age $g > \hat{g}$ holds and zero otherwise.[5] We define the old-age-specific and the uniform components of income risk by rearranging the terms in Equation (5):

$$\sigma^2_{t,g} = \sigma^2_t + \sigma^2_{y,t} + I_{g>\hat{g}} \cdot (\sigma^2_{o,t} - \sigma^2_{y,t}) = \tilde{\sigma}^2_t + I_{g>\hat{g}} \cdot \Delta\sigma^2_{yo,t} \quad (6)$$

Equation (6) states that the income risk of a household of age is a sum of a uniform risk component, $\tilde{\sigma}^2_t$, and the old-age-specific risk component $\Delta\sigma^2_{yo,t}$. While the uniform component affects the risk of all age groups, the old-age-specific component affects only the risk faced by workers older than $\hat{g}$. Note that the size of old-age-specific component at time t depends on the gap between "old" and "young" risk $\Delta\sigma^2_{yo,t} = \sigma^2_{o,t} - \sigma^2_{y,t}$. The "old"-"young" risk gap $\Delta\sigma^2_{yo,t}$ captures the additional risk faced by older workers compared to the younger workers, and, hence, can be positive or negative. The old-age-specific risk component $\Delta\sigma^2_{yo,t}$ follows an AR(1) process:[6]

$$\Delta\sigma^2_{yo,t} = (1-\rho) \cdot \Delta\bar{\sigma}^2_{yo} + \rho \cdot \Delta\sigma^2_{yo,t-1} + \Delta\epsilon_{yo,t}, \quad \Delta\epsilon_{yo,t} \sim F_{yo} \quad (7)$$

The distribution of old-age-specific innovation $\Delta\epsilon_{yo,t}$ is such that $E(\Delta\epsilon_{yo,t}) = 0$, $E(\Delta\epsilon_{yo,t})^2 = \sigma^2$ and has unbounded support. We refer to $\Delta\epsilon_{yo,t}$ as the old-age-specific income risk shock. A positive old-age-specific risk shock captures the unexpected increase in the difference between "old" and "young" workers' risk, potentially stemming from either an increase in "old" risk *or* or a decrease in "young" risk. As we show below, the patterns in the data suggest that this shock should be interpreted as a positive shock to the risk faced by older workers rather than a negative shock to the risk faced by younger workers.

[5] Note that if we let $\sigma^2_y = \sigma^2_o = 0$, our model closely tracks the specification of Bayer et al. (2019), with the "common" component being the only determinant of income risk.

[6] We test the significance of higher-order lags and find that they are insignificant.

3.3 Estimation

For the baseline estimation, we assume a fully persistent process for permanent income such that $\rho_h = 1$.[7] For each group of households defined by age g and time t, we compute the cross-sectional contemporaneous and lagged variance of income residual distribution from Equation (2):

$$\phi_1(t,g) = E(u_{i,t}^2 | i \in (t,g)) = \sigma_\mu^2 + (1+\rho_\tau)\sigma_\tau^2 + \sigma_h^2(t,g), \tag{8}$$

$$\phi_2(t,g) = E(u_{i,t-1}^2 | i \in (t,g)) = \sigma_\mu^2 + (1+\rho_\tau)\sigma_\tau^2 + \sigma_h^2(t-1,g-1) \tag{9}$$

Then the change in ex-post income residual variance captures information about the ex-ante income risk faced by households of age g at time t[8]

$$\Delta\phi(t,g) = \phi_1(t,g) - \phi_2(t,g) = \sigma_{t,g}^2 = \tilde{\sigma}_t^2 + I_{g>\hat{g}} \cdot \Delta\sigma_{yo,t}^2 \tag{10}$$

We construct the time-age panel of empirical second-moment differences $\Delta\phi(t,g)$ from the SIPP data (see estimation procedure below), capturing the risk faced by each age group at each point in time. Then, informed by Equation (10), we construct the regression specification that allows us to decompose income risk into the contributions of the uniform and old-age-specific factors:

$$\Delta\phi(t,g) = b_0 + \sum_{t=0}^{T} b_t \cdot I_t + \sum_{t=0}^{T} \gamma_t \cdot I_t \cdot I_{g>\hat{g}} + v_{t,g} \tag{11}$$

where I_t is a time dummy and $I_{g>\hat{g}}$ is an “old” age dummy. In this specification, b_t captures the time t risk, uniform across all age groups, and γ_t captures the old-age-

[7] The typical finding in the literature is that the autocorrelation of permanent income is quite high, exceeding 0.95 at the quarterly frequency. For example, see Floden and Lindé (2001); Storesletten et al. (2001, 2004); Bayer et al. (2019). We show that our results are robust to changes in the persistence parameter.

[8] Note that taking the difference of these two moments removes the cohort-specific effect and the transitory income risk.

specific component of income risk, which is the main object of our analysis.

Estimation procedure. The starting point of our estimation procedure is to compute the individual labor income residuals. We remove the deterministic part of labor income, $f(X_{i,t})$, in Equation (1) using the OLS procedure. The variables included in $X_{i,t}$ are: year and quarter of observation (to control for inflation and other common time trends in labor income), age, race, household size, education level of the head of household, and interaction terms of education level with age and age squared (to control for changes in the marginal return to education with experience).

The income residual estimate, $\hat{u}_{i,t}$, is the variation in labor income not explained by the above-mentioned factors. We compute the income residual and its first lag for each household i and each time t. Then, we compute the empirical counterpart of our theoretical moments provided by Equations (8)-(9). For this, we group households into time-age groups (t, g) and calculate the empirical variance of income residuals for each time-age group (t, g) and its lag: $Var(\hat{u}_{i,t}|i \in (t, g))$, $Var(\hat{u}_{i,t-1}|i \in (t, g))$. The empirical moment difference for each time-age group corresponding to our risk measure for workers of age g at time t is then computed as $\Delta\phi(t, g) = Var(\hat{u}_{i,t}|i \in (t, g)) - Var(\hat{u}_{i,t-1}|i \in (t, g))$.[9]

Finally, to estimate the old-age-specific income risk, we fit Equation (11) to our empirical moment differences. For this purpose, we first establish the age threshold $\hat{g}$ separating the older workers from the rest. We choose the age threshold to maximize the average absolute difference between the "young" and "old" risk. Then estimating Equation (11) yields the old-age-specific risk component given by γ_t, and the corresponding uniform risk component given by b_t (both measured up to a constant b_0).[10]

[9] In the constructed time-age panel of risk measures $\Delta\phi(t, g)$, we set the two outliers corresponding to 2000Q4 and 2001Q1 to missing values as these periods mark the start of the 2001 panel and differ substantially from the last values obtained from the previous 1998 panel (see Figure A.1 in Appendix A). Then we impute missing moment differences along the time dimension for each age group. In the robustness exercises, we demonstrate that the results remain robust to these modifications.

[10] We build confidence bands around the age-specific income risk estimates using $B = 1000$ bootstrap iterations. At

Finally, we fit an AR(1) process to each of these estimated risk components to obtain the corresponding risk innovations, which we refer to as the old-age-specific income risk shock and the uniform income risk shock.

3.4 Results

Now, we present our estimation results. First, we examine the extent to which business cycle fluctuations in income risk are age-specific. Then we report the estimated age-specific consumption response to the old-age-specific income risk shock and compare it with the response to a uniform risk shock.

3.4.1 Age-specific income risk

Figure 1 (left panel) plots the "young" and "old" workers' income risk over time for the estimated age threshold between young and old equal $\hat{g} = 45$.[11] We observe that the income risk faced by older workers is almost always higher than the risk faced by younger workers. Moreover, the old-age-specific component of risk (the gap between young and old risk, given by γ_t in our regression) is statistically different from zero in many periods (right panel).

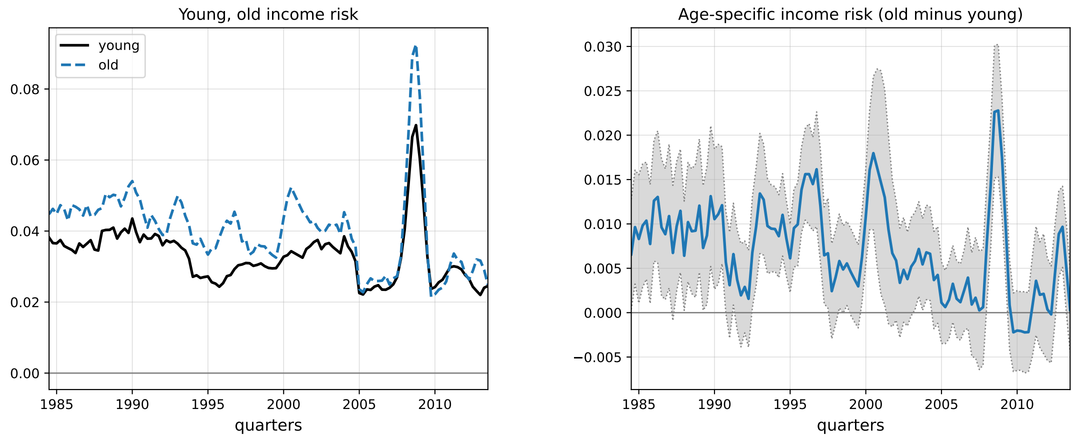
3.4.2 Age-specific and uniform risk shocks

Figure 2 panel (a) plots the estimated realizations of the old-age-specific and the uniform income risk shocks; we observe that these shocks have comparable magnitudes of volatility. Note that the uniform risk shock is similar in spirit to the innovation estimated in Bayer et al. (2019) (in the sense that it affects the risk of all age groups uniformly), while the old-age-specific risk shock constitutes a novel contribution of

every bootstrap iteration, we resample the income residuals for each time-age group.

[11] In Appendix A we plot the age profile of income risk volatility over time (Figure A.2, left panel). This profile graphically reveals that income risk of workers older than 45 is more volatile over time, compared to the risk faced by younger workers. We also demonstrate that our main result is nevertheless robust to the shift in this age threshold.

Figure 1: Income risk over time: young vs. old



Left panel plots the risk faced by people younger than $\hat{g}$ $(b_0 + b_t)$ and people older than $\hat{g}$ $(b_0 + b_t + \gamma_t)$ for $\hat{g} = 45$. The right panel plots the estimated sequence of age-specific income risk effects γ_t with one standard deviation bootstrapped confidence bands.

the present analysis. The estimated persistence of the old-age-specific income risk is $\rho = 0.84$, which is in line with the corresponding estimates of Bayer et al. (2019) for the uniform income risk shock. The standard deviation is $Std(\Delta\epsilon_{yo,t}) = 0.0033$.

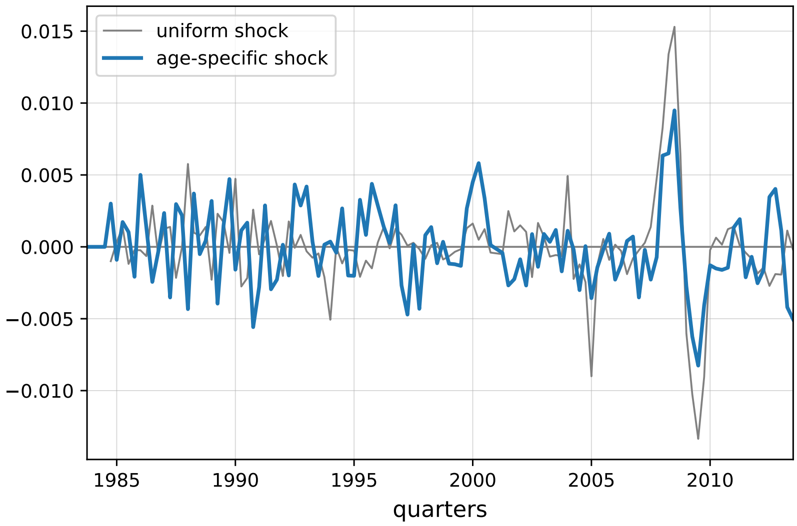
The conceptual difference between the uniform risk shock and the old-age-specific risk shock lies in their effects on income risk across age groups. Figure 2 panel (b) plots the contemporaneous change in income risk across age groups following each shock. While the old-age-specific income risk shock does not affect the income risk of younger workers but significantly increases the income risk of workers above 45, the uniform risk shock increases the income risk of all age groups uniformly.

3.4.3 Effect of risk shocks on consumption

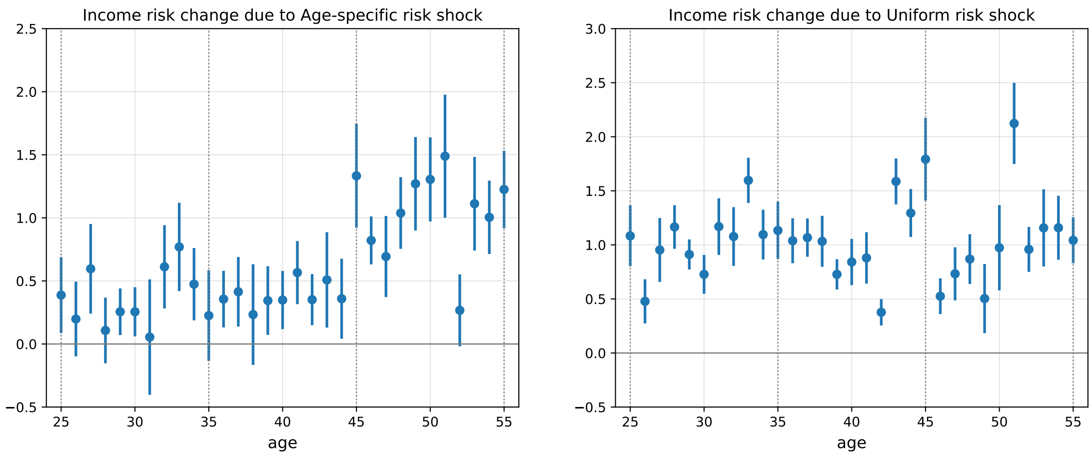
Next, we estimate the consumption response to the old-age-specific income risk shock, as well as to the uniform risk shock. We begin by considering consumption for three age groups: 25-34, 35-44, and 45-55 years, referred to as younger, middle, and older workers. We use the local projection method (Jordà, 2005) to estimate the impulse

Figure 2: Age-specific and uniform risk shock

(a) Shock realizations over time



(b) Income risk change due to shocks



The change in income risk after the old-age-specific income risk shock (left panel) and the uniform risk shock (right panel). The vertical bars correspond to 66% asymptotic (Newey and West (1987) robust) confidence bands.

response functions of age-specific consumption to the old-age-specific and the uniform risk shocks. In the baseline estimation, we control for three lags of the dependent variable and a time trend.[12] Asymptotic confidence intervals are constructed using Newey and West (1987) robust standard errors.

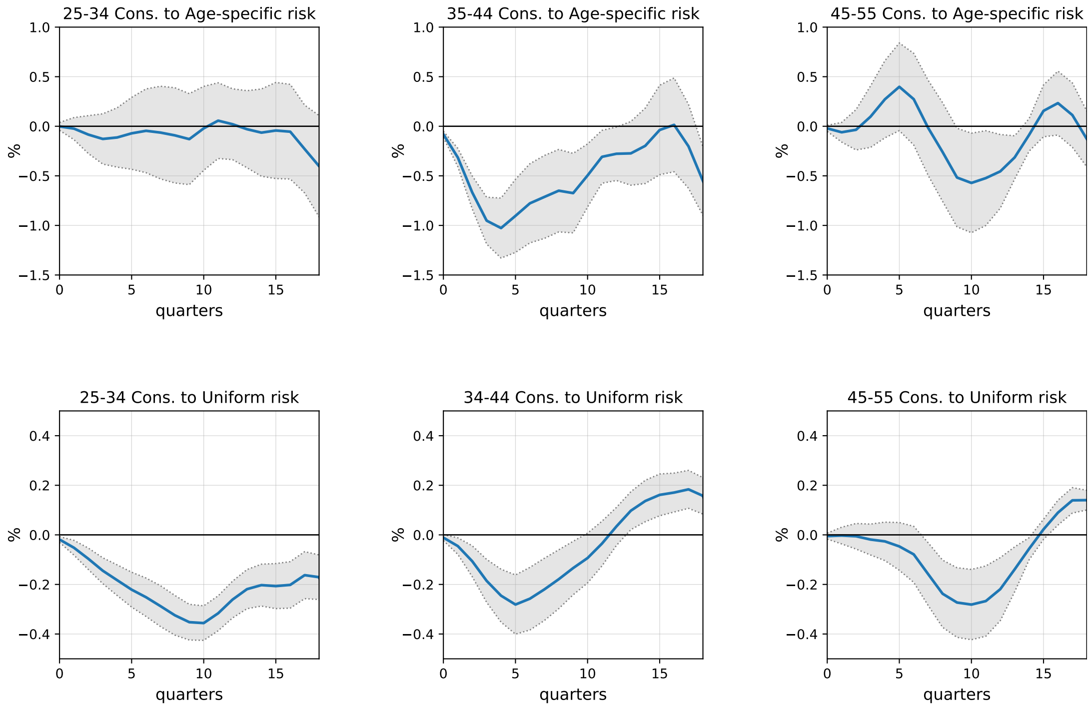
Figure 3 reports our impulse response estimates. The top row of Figure 3 reports the age-specific consumption response to the old-age-specific income risk shock. We observe that young workers' consumption does not respond to this shock, and the response of older workers' consumption is quite moderate. At the same time, middle-aged consumption experiences the most pronounced drop in response to the old-age-specific risk shock, even though the income risk of this age group is not directly affected by the old-age-specific risk shock.[13] One possible explanation for this pattern is that middle-aged individuals are particularly sensitive to future income stability as they approach the later stages of their working lives. For these individuals, increased risk facing older workers may serve as a forward-looking signal, raising concerns about their own financial security in the near future. This anticipatory response aligns with the concept of "news shocks," where information about future risks affects present consumption behavior. As a result, old-age-specific risks affect not only the targeted age group but also spill over to other cohorts. Our findings suggest that age-targeted policies aimed at stabilizing income for older workers might indirectly influence the consumption patterns of middle-aged workers as well, contributing to broader economic stability.

Additionally, we estimate the age-specific consumption response to a uniform risk shock (bottom row of Figure 3). In response to a positive uniform risk shock, young consumption exhibits the strongest decline, while middle-aged and old-aged consump-

[12] In robustness exercises, we demonstrate that our results are robust to adding more control variables such as lagged shocks, aggregate output, policy rate, and an extended number of lags of the dependent and other variables. See Appendix A for the results of these robustness exercises.

[13] As evident in the panel (b) of Figure 2.

Figure 3: Consumption response to risk shocks



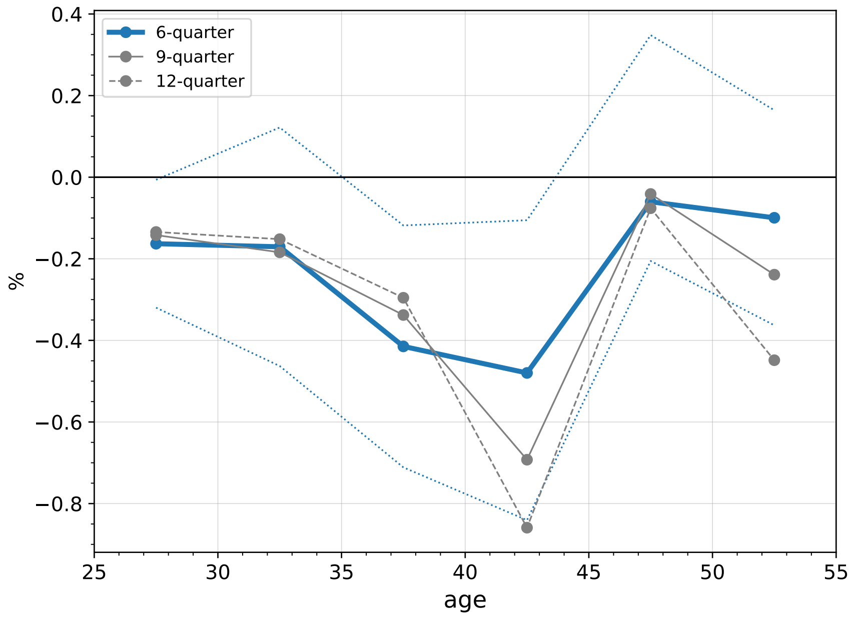
This figure plots the percentage response to 1 standard deviation age-specific risk shock (top row), and 1 standard deviation uniform risk shock (bottom row). The grey areas denote the 66% confidence bands.

tion responds with a more moderate drop. This sharply contrasts with the impact of the old-age-specific risk shock across age groups.

Hence, our old-age-specific income risk shock has a unique feature in that it induces disproportionately more consumption fluctuations in middle-aged workers.[14] In Appendix A, we also report the response of some key macroeconomic variables to our old-age-specific risk shock – while this shock is generally contractionary, it does not induce any significant policy response in the nominal interest rate.

[14] In Appendix A we plot the age profile of income and consumption business cycle volatility, showing that the middle age consumption exhibits the most volatility over time while middle-aged income is the least volatile. Hence, our shock can potentially contribute to explaining the high relative volatility of middle-aged consumption.

Figure 4: Age-profile of cumulative consumption response to age-specific risk shock



This figure plots the cumulative consumption response in each 5-year age group (normalized by the corresponding number of quarters). Blue dotted lines plot 66% confidence bands for the 6-quarter profile.

3.4.4 Age profile of consumption response to old-age-specific risk shock

Next, we construct the age profile of consumption response to the old-age-specific risk shock for more disaggregated consumption series. For this, we consider age-specific consumption for six age groups spanning 5 years each between 25 and 55 years (25-29, 30-34, 35-39, 40-44, 45-49, 50-55). For each age group, we compute the cumulative change in consumption within six, nine, and twelve quarters after the shock. Figure 4 plots the resulting profiles (each cumulative response is divided by the corresponding number of quarters). The age profile of consumption response to the old-age-specific risk shock is U-shaped, with workers in the 40-44 years group being the most responsive to the shock. In the Appendix A, we report the underlying dynamic impulse responses for each of these six age groups.

3.5 Summary of robustness checks

Now we briefly outline the robustness checks that examine various aspects of our estimation process. The corresponding results are reported in Appendix A.

Persistence of permanent income. In the baseline estimation, we explicitly assume that the permanent income component follows a random walk ($\rho_h = 1$). Usually, empirical papers report somewhat lower income persistence. For example, Bayer et al. (2019) report quarterly persistence of 0.98, while Floden and Lindé (2001) document yearly persistence, which corresponds to about 0.977 at the quarterly frequency. In A, we reestimate our old-age-specific risk shocks under the assumptions of a smaller value of ρ_h and demonstrate that our results are robust to this modification.

Interpolation. In the baseline estimation, we impute missing values in the empirical time-age panel of income risk measures. As a robustness check, we shift the imputation of missing information from the initial risk measures to a later stage of estimation where we estimate the income risk components. Specifically, we estimate the old-age-specific risk component in a separate regression for each quarter (instead of relying on time-fixed effects), excluding dates for which the moment differences are missing. Then, we impute missing values for the estimated age-specific risk component. Appendix A shows that our results are robust to this modification.

Outliers. In the baseline, we treated 2000Q4 and 2001Q4 as outliers. If we include these observations, the baseline result still holds: middle-aged consumption remains the most responsive to the old-age-specific risk shock.

Local projection controls. We check the robustness of our results by adding more control variables. We extend the number of lags of control variables to six and include lags of both shocks, aggregate output, and policy rate. Our results are robust to this modification.

Age threshold. We also reestimate the age-specific risk for a different choice of threshold separating young and old workers. Instead of 45 years, we consider alternative values of 40 and 50. The choice $\hat{g} = 50$ does not significantly alter our consumption responses. For $\hat{g} = 40$, the response of the middle-aged group is even stronger, as the immediate risk faced by this group is now included in the old-age risk innovation. The response of the young and old groups' consumption remains muted, similar to the baseline result.

Smooth weights. An alternative to picking the strict cutoff age is to assign a smooth weight $F(g)$ to the old age component in the income risk faced by each age g group, such that $F(g)$ increases with age. We take the logistic function $F(g) = \frac{1}{1+e^{-\xi(g-\hat{g})}}$ with $\xi = 1$ and $\hat{g} = 45$. Our results remain robust to this modification. Note, that our baseline model with strict cutoff age obtains when $\xi \to \infty$.

4 Life-cycle model

In this section, we evaluate the ability of a standard life-cycle model featuring a precautionary saving motive to reproduce our empirical findings on the U-shaped age profile of consumption response to the old-age-specific income risk shock. To this end, we consider a heterogeneous agent Huggett-type overlapping generations (OLG) framework. The OLG structure allows us to model income risk variations specific to each age group. The heterogeneous-agent structure is particularly well-suited for our focus on precautionary savings, as the model incorporates differences in marginal propensities to consume (MPC) and prudence across agents, reflecting real-world consumption behavior more accurately than a representative agent model would. Furthermore, the Huggett-type model offers a balance between realism and tractability, providing a clear framework to isolate the effects of age-specific income risk without introducing unnecessary complexity from multi-asset structures. This design allows us to focus directly on consumption response to income risk fluctuations.

Within the model framework, we derive an approximate analytical response of age-specific consumption to an arbitrary income risk shock, which nests the old-age-specific and uniform income risk shocks as special cases. This analytical response is expressed using the distributions of two household characteristics: the marginal propensity to consume out of labor income and labor income prudence. The novelty of our labor income MPC and prudence concepts lies in their relation to changes in the *labor* income as opposed to the lifetime income. In the presence of retirement, permanent labor income generally differs from permanent lifetime income because labor income stops upon retirement, which typically occurs long before death.

To compute realistic distributions of the labor income MPC and prudence measures, we calibrate our model to the U.S., taking into account stationary age profiles of income level and income risk, as well as survival rates. Solving the model results in a realistic stationary distribution of households across income, wealth, and age. Using the distributions of labor income MPC and prudence produced by the model, we then compute the consumption response to the old-age-specific risk shock and the uniform risk shock implied by consumption theory. Our results suggest that the decreasing age and wealth profiles of labor income MPC and labor income prudence are crucial for generating the empirically plausible U-shaped profile of consumption response to the old-age-specific risk shock.

4.1 Model description

Our model is a heterogeneous-agent OLG model. The model economy operates in continuous time and runs forever. At each point in time, the economy is populated by households of different ages, ranging from the youngest to the oldest. A period in the model corresponds to a year, meaning that all the flow variables are measured "per year".

Population structure. Each period, a share of households dies and exits the economy. The probability of death depends on age. At the same time, new households of the youngest age enter the economy. Every generation in the model is uniquely defined by its birth date $\underline{t}$. Each household within a generation can reach a maximum age of $\bar{g}$; thus, any household born at $\underline{t}$ passes away with certainty at $\underline{t}+\bar{g}$. Throughout their life, households face a probability of death. The probability of surviving until age g ($g \in [0,\ \bar{g}]$) is $p(g)$, such that $p(0) = 1$, $p(\bar{g}) = 0$, and $p'(g) \leq 0$.

Households. Throughout their lives, households work, consume, and make savings. They inelastically supply one unit of labor per period and receive a stream of risky labor income. Upon reaching retirement age, households stop working and begin receiving the flow of riskless retirement benefits. Households choose consumption and savings paths that maximize their expected discounted utility. The problem for a household born at date $\underline{t}$ and holding asset a_t and having income y_t is

$$V^{\underline{t}}(a_t, y_t) = \max_{c_h, a_h} E_t \int_t^{\underline{t}+\bar{g}} e^{-\rho_h(h-t)} u(c_h) dh + \bar{u}(a_{\underline{t}+\bar{g}}) \tag{12}$$

$$s.t.\ \dot{a}_t = r \cdot a_t - c_t + I_{\{g<R\}} \cdot y_t + I_{\{g \geq R\}} \cdot b \tag{13}$$

$$a_t \geq -d, \quad a_{\underline{t}+\bar{g}} \geq 0 \tag{14}$$

where c_h is consumption, y_t is the per-period labor income, b is retirement benefit, $g = t - \underline{t}$ denotes the household's age, R is the retirement age. $I_{\{.\}}$ is an indicator function that takes the value of 1 if the condition in brackets is true, d is the borrowing constraint, and $a_{\underline{t}+\bar{g}} \geq 0$ is the terminal condition indicating that a household cannot pass away with debt. The instantaneous utility function $u(.)$ is such that $u' > 0$ and $u'' < 0$. The terminal utility $\bar{u}(.)$ represents utility from dying with wealth (bequest motive). Finally, the discount rate ρ_t consists of a subjective discount factor ρ and a conditional survival probability, so that $e^{-\rho_h(h-t)} = \frac{p(h-\underline{t})}{p(t-\underline{t})} \cdot e^{-\rho(h-t)}$, where $\frac{p(h-\underline{t})}{p(t-\underline{t})}$ is the probability of surviving until date h (or age $h - \underline{t}$), conditional on being alive at date t (at age $t - \underline{t}$), where $h \geq t$.

The labor income depends on the age profile of workers' productivity χ_g and age-specific stochastic process $z_{t,g}$ such that:

$$y_t = \bar{y} \cdot e^{\chi_g \cdot z_{t,g}} \tag{15}$$

$$dz_{t,g} = -\theta z_{t,g} dt + \sigma_g dW_t, \quad dW_t \sim \mathcal{N}(0, \sqrt{dt}) \tag{16}$$

where χ_g is a deterministic productivity component that depends only on age g, $z_{t,g}$ is an idiosyncratic stochastic part of the labor income following the Ornstein-Uhlenbeck process with drift $-\theta$ and age-dependent diffusion σ_g. The labor income process is a continuous-time version of the dynamic labor income model used in the empirical section. The two age-dependent parameters χ_g and σ_g allow us to realistically calibrate the stationary age profiles of labor income and labor income risk.

4.2 Consumption response to risk shocks

Now we turn to characterizing an analytical response of age-specific consumption to an arbitrary change in income risk. Analytical characterization is feasible as long as the borrowing constraint is sufficiently loose for the household's Euler equation to hold with equality. Hence, we assume that d corresponds to the natural borrowing limit. We derive the age-specific consumption response to current and expected future risk innovations, by expressing the sensitivity to risk in terms of the second moment of the joint distribution of two novel household characteristics: marginal propensity to consume out of permanent *labor* income, and prudence to *labor* income risk.

In the presence of retirement, when a person stops working long before her death, our notions of MPC and prudence differ from the standard MPC and prudence definitions. The standard MPC relates to either a response of consumption to a one-time transfer (transitory income MPC) or, alternatively, to a change in permanent income (permanent income MPC). In contrast, our MPC notion pertains specifically to the change in permanent *labor* income, which is not strictly permanent as it ends with retirement. Similarly, our prudence measure captures the sensitivity to permanent labor income risk, as opposed to lifetime permanent income risk. Next, we provide the formal definitions of our labor income MPC and prudence measures.

Definition 1 (Labor income MPC)**.** The marginal propensity to consume out of permanent labor income measures an increase in consumption in response to a 1$ increase in permanent labor income.

Definition 2 (Labor income prudence)**.** Prudence to permanent labor income risk measures the dollar increase in permanent labor income required to keep consumption unchanged when labor income risk, measured as the standard deviation of the permanent labor income innovation, increases by 1$.[15]

Next, we analytically characterize the optimal consumption function. The following proposition describes the optimal consumption policy in terms of labor income MPC and prudence.

Proposition 1. *Consider a working-age household of age g solving the optimization problem (12)-(14). Its optimal consumption function is*

$$c_g = \frac{L_g}{X_g} - E_g \int_g^R e^{-r(h-g)} \cdot \frac{X_h}{X_g} \cdot \mu_h \nu_h \sigma_h^2 dh + E_t \int_g^{\bar{g}} e^{-r(h-g)} \cdot \frac{X_h}{X_g} \cdot I_h dh \tag{17}$$

where $L_g = E_g \left\{ a_g + \int_g^R e^{-r(h-g)} E_g y_s ds + \int_R^{\bar{g}} e^{-r(h-g)} b ds \right\}$ *is the expected lifetime wealth,* μ_h *is the labor income MPC,* ν_h *is the labor income prudence,* $X_h = \int_h^{\bar{g}} e^{-r(s-h)} ds$ *is the inverse of the standard cash-in-hand MPC,* I_h *is the impatience term.*

See Appendix B for the derivations.

[15] Note that our definition of prudence is done in the spirit of the classical consumption literature Kimball (1990) and somewhat differs from the alternative definition based on the ratio of the third derivative over the second derivative of the utility function, as sometimes found in the literature.

The optimal consumption function in Proposition 1 incorporates three terms. The first term indicates that consumption depends on expected lifetime wealth. The second term accounts for precautionary saving, representing how consumption depends on the expected income risk path faced by the household between the current age g and retirement R. The sensitivity of consumption to income risk is determined by the product of the labor income MPC μ and labor income prudence ν. Finally, the third term captures the impatience arising because the interest rate does not generally equal the discount factor.

Recall that labor income prudence, ν, measures the permanent labor income compensation required to keep consumption unchanged when the corresponding labor income risk changes by 1\$. Hence, ν measures the strength of the precautionary motive in terms of the equivalent permanent labor income change. At the same time, labor income MPC μ determines the consumption sensitivity to this equivalent income change. Therefore, the product $\mu \cdot \nu$ naturally determines the sensitivity of consumption to a change in income risk.

Before proceeding, let us discuss in more detail the difference between our measure of the labor income MPC and the standard permanent income MPC for the unconstrained household within the context of our model. From the first term of Equation (17), it is clear that the MPC out of fully permanent income (income change relates to both labor income and retirement benefit) is 1.[16] At the same time, the MPC out of permanent labor income (with income change related only to the working age) is approximately $\mu_t \approx \frac{R-g}{\bar{g}-g}$, which is the ratio between the time left to retirement and the time left to death.[17] Hence, the MPC out of permanent labor income decreases with age even for an unconstrained household, in contrast to the standard permanent

[16]To see this, consider a variation in each period income dy. The corresponding change in lifetime wealth is $dL_g = (\int_g^R e^{-r(h-g)}ds + \int_R^{\bar{g}} e^{-r(h-g)}ds)dy = X_g \cdot dy$. Hence, the corresponding change in consumption is $dc = dy$

[17]To see this, consider a change in labor income dy. The corresponding change in consumption is $dc = \frac{dL_g}{X_g} = \frac{\int_g^R e^{-r(h-g)}ds}{X_g}dy = \frac{1-e^{-r(R-g)}}{1-e^{-r(\bar{g}-g)}}dy \approx \frac{R-g}{\bar{g}-g}dy$ for small r.

income MPC.

Now we turn to the effect of an arbitrary income risk shock on age-specific consumption. We define age-specific consumption as the weighted average consumption within an age group. We consider an arbitrary change in the expected income risk path faced by a particular age group after the shock, denoted by $\{\Delta\sigma_h^2\}_{h\geq t}$. The following Corollary characterizes the corresponding age-specific consumption response:

Corollary 1. *Consider all individuals within age group G and a change in the expected income risk path faced by this group, denoted by $\{\Delta\sigma_h^2\}_{h\geq t}$. The approximate age-specific consumption response to this change around the zero-risk steady-state is*

$$\Delta C_t^G = \int_G \Delta c_t^i \approx -\int_t^{\bar{t}} e^{-(r+\frac{1}{\bar{t}-t})(h-t)} \cdot E_G[\mu_h^i \nu_h^i] \cdot \Delta\sigma_h^2 dh \tag{18}$$

where ΔC_t^G denotes the average consumption change in group G, i is the index of a household within G, and $E_G[\mu_h^i \nu_h^i]$ is the cross-sectional average of the product of labor income MPC and prudence in group G.

See Appendix B for the derivation.

Corollary 1 establishes the main theoretical result of this section, showing that the magnitude of the age-specific consumption response to an income risk shock depends on the distribution of labor income MPC and prudence within each age group.

Since prudence and MPC generally depend on the level of wealth and the possibility of hitting the borrowing constraint, evaluating the aggregate product of MPC and prudence necessitates a realistic distribution of wealth within each age group. Therefore, we resort to numerical computation of the model-implied distributions of labor income MPC and prudence across age groups and wealth levels within the heterogeneous-agent OLG model, calibrated for the U.S. We then employ the model-implied distributions of these MPC and prudence to construct the quantitative response of consumption to the old-age-specific and uniform income risk shocks from the Corollary 1.

4.3 Model calibration, solution, and properties

Now we describe the calibration of the model which we subsequently use to construct the labor income MPC and prudence distributions. Then we briefly discuss the important properties of the model.

4.3.1 Calibration and solution

The model is calibrated in line with the U.S. data.

Population. Each household reaches economic maturity at the age of 25. Limiting the minimum age in the economy to 25 years allows us to abstract from heterogeneity in education choice. The maximum duration of economic life is 60 periods ($\bar{g} = 60$), and no agent survives beyond age 84. Upon reaching 63 years ($R = 38$), agents receive a retirement benefit for a maximum of 22 years ($\bar{g} - R = 22$). We calibrate the yearly mortality rates using data from the National Vital Statistics Report published by the National Center for Health Statistics (2006).

Income and Social Security. We calibrate two groups of parameters related to the labor income process of households: a deterministic age-specific productivity profile and parameters governing the stochastic component of the income process.

Using the SIPP income data, we construct the deterministic age-specific productivity measure χ_g. To construct the deterministic age-specific component, we fit a cubic regression of the logarithm of quarterly income on age. The resulting profile exhibits an inverse U-shaped form, with peak productivity occurring after approximately 20 years of labor market experience.[18] The life-cycle component of productivity χ_g is set to zero for households older than 62 years, corresponding to retirement age. See Appendix B for a visual comparison of the data and the model income profiles.

[18] The exact coefficients are $a_3 = 5.635 \cdot 10^{-7}$, $a_2 = -1.113 \cdot 10^{-3}$, $a_1 = 9.476 \cdot 10^{-2}$, $a_0 = 6.702$

The US Social Security system provides retirement benefits based on the level of pre-retirement earnings. We adopt the approach of Guvenen and Smith (2014) to mimic the main features of this system. We model a pension benefit as a function of labor income in the last working year (a proxy for lifetime earnings) and the average wage bill in the economy.

Income process. The stochastic component of productivity is the Ornstein – Uhlenbeck process with drift $-\theta$ and age-specific diffusion σ_g. We set θ to 0.09 to target the value for the log income autocorrelation of 0.91 (Floden and Lindé, 2001). To build the age profile of income risk σ_g, we fit a cubic regression to the average of the income risk measure that we previously constructed from the SIPP data. Our risk profile has a U-shape form, similar to Karahan and Ozkan (2013). See Appendix B for a visual plot of the empirical and model risk profiles.

Preferences and Constraints. Households have a standard time-separable CRRA utility function with the instantaneous utility given by $u = \frac{c^{1-\sigma}}{1-\sigma}$, where σ is the constant relative risk aversion parameter. Relative risk aversion affects the intensity of the precautionary motive. We set the utility parameter σ to 3, a common value in the heterogeneous-agent literature. Our choice of utility function results in a level of absolute prudence that decreases strongly with wealth - a desirable feature of precautionary saving models (Kimball, 1990).

Our calibration strategy of the interest rate r and borrowing constraint d is based on Kaplan and Violante (2014). We set $r = 1.67\%$ and the debt limit is determined as 74% of households' mean annual labor income. We calibrate the discount factor $\rho = 0.055$ to replicate the empirical ratio of median labor income to median net worth. In the Survey of Consumer Finance (SCF), the median net worth is approximately twice as large as the before-tax family income. Considering that we are dealing with after-tax income within the model, we match a somewhat higher median net worth-to-income ratio of 2.8.

Initial Distribution and Bequest. We initialize the model by setting the earning-wealth distribution for 25-year-old households. For this purpose, we collect the data from the 2013 SCF for households aged 20-24 years. We exclude the top 5% wealthiest households and divide the remaining sample into groups corresponding to bins based on wealth points on our grid. We assume household earnings are uniformly distributed within each asset bin, given that the empirical correlation between labor income and wealth for the 25-year-old age group is close to zero.

We calibrate the terminal condition to reflect the intended bequest motive. The utility from bequest takes a CRRA form for positive terminal values of assets. For negative values of assets, the terminal condition ensures that no agent dies with debt. The intended bequest, which we aim to match, represents approximately 1% of the economy's aggregate consumption.

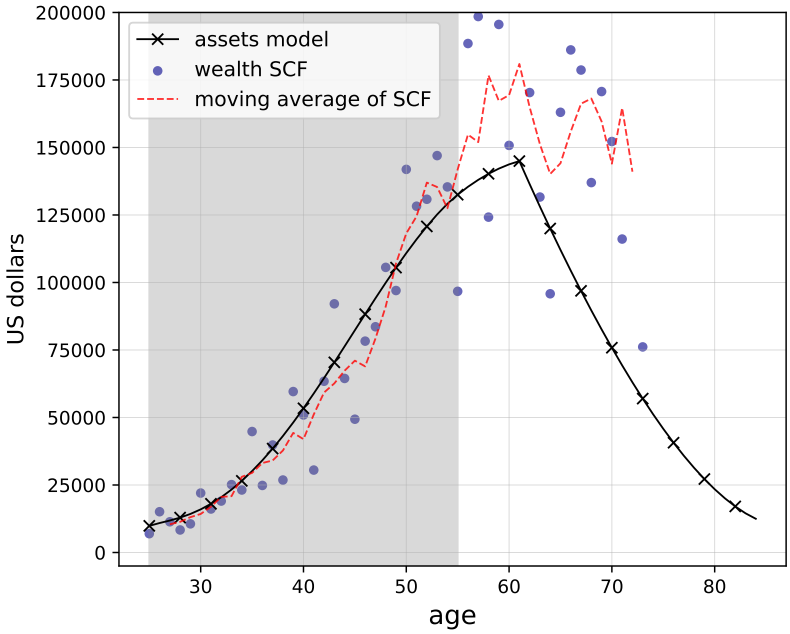
Finally, we formulate the model in terms of the Hamilton-Jacobi-Bellman equation for the household problem and a corresponding Kolmogorov forward equation for the evolution of income and wealth distribution. We solve the model using a finite difference scheme (Achdou et al., 2022). For the recursive formulation and solution details, see Appendix B.

4.3.2 Model properties

Let us briefly examine the key features of the model. Figure 5a plots the age profile of asset accumulation in the model and compares it to the corresponding empirical profile. For the empirical profile, we use data from the Survey of Consumer Finances (SCF), specifically from the wave 2013. We interpret the net worth of households as the empirical counterpart of asset holdings in our model. Net worth includes financial and non-financial liquid assets of households minus total debt. We focus on the median asset holdings by age. The asset accumulation profile in the model aligns with the observed data for the working-age population we are considering –

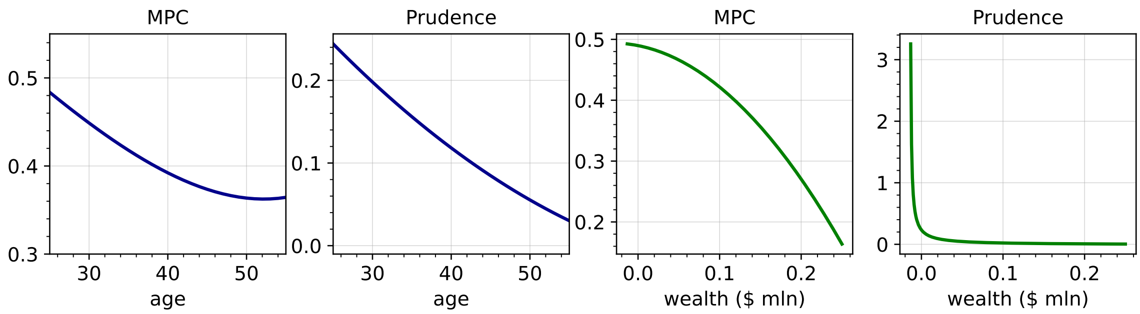
Figure 5: Main properties of the model

(a) Life-cycle asset profile: model vs. data



The plot displays the model's asset profile by age alongside the corresponding asset profile from the Survey of Consumer Finances. The red dashed line represents the moving average (over five periods) of the SCF data points. The grey area highlights the age range we are focused on.

(b) Age and wealth profiles of labor income MPC and prudence



This figure illustrates the model-generated mean MPC and prudence across age and wealth dimensions. The MPC measures the increase in consumption in response to a $1 permanent increase in labor income. Prudence measures the decrease in permanent labor income equivalent to a $100 increase in labor income risk.

from 25 to 55 years old. After age 60, households in the model begin to decumulate assets, whereas we do not observe this pattern in the data. This discrepancy arises because we model post-retirement household behavior in a reasonably stylized way, abstracting from potential post-retirement risks (such as health shocks) that could inhibit asset decumulation among older individuals. This feature, however, does not affect our analysis as we concentrate on households younger than 55.

As we use the model to extract the numerical distributions of the labor income MPC and prudence, let us briefly look into their age and wealth profiles. Figure 5b shows that, on average, the labor income MPC and prudence decrease in age and wealth. Given the obvious correlation between age and wealth, it is generally difficult to identify their separate effect on MPC and prudence; a similar point was made in Fagereng et al. (2021) – the quasi-experimental study of transitory income MPC. However, as we showed in Section 4.2, the labor income MPC of an unconstrained household decreases with age for any given level of wealth. This implies that the decreasing age profile of labor income MPC (and potentially prudence) is at least partially shaped by age rather than solely by wealth level.

4.4 Results

Now we turn to evaluating the ability of our analytical result in Corollary 1 to reproduce the empirical U-shaped age profile of consumption response to the old-age-specific labor income risk shock. According to Corollary 1, the age-specific consumption response to a risk shock is driven by the sequence of expected income risk changes $\{\Delta\sigma_h^2\}_{h\geq t}$ induced by the shock, and the distribution of labor income MPC μ_h^i and prudence ν_h^i across households within each age group. We employ the numerical distributions of labor income MPC and prudence generated by our quantitative model. We calibrate the sequence of risk changes to align with the empirical evidence of Section 3. Specifically, we assume that at time $t = 0$ the old-age risk component is perturbed with one standard deviation shock, followed by a gradual decay in the subsequent periods according to Equation 7; the shock size and persistence are parameterized according to their estimated values. Then the sequence of expected income risk changes $\{\Delta\sigma_h^2\}_{h\geq t}$ faced by each age group after the shock is computed from Equation 6.

Figure 6a ("full effect") shows the age profile of consumption response to the old-age-specific risk shock (cumulative over six quarters). We see that the model successfully reproduces the empirical U-shaped profile. Consistent with our empirical evidence, the age group from 40 to 44 experiences the strongest consumption decline, even though the old-age-specific risk shock affects the income risk only for workers aged 45 and older.

To quantify the contributions of heterogeneous risk versus heterogeneous sensitivity to risk we also compute two counterfactual profiles: 1) S_1 with constant labor income MPC and prudence across households, to highlight the role of heterogeneous risk profiles faced by different age groups after the shock, 2) S_2 which accounts for age heterogeneity in labor income MPC and prudence, but not wealth heterogeneity, highlighting the separate contribution of age dimension of MPC and prudence heterogeneity leading to the differences in risk sensitivity across age groups. Next, we describe each counterfactual in turn.

Role of income risk heterogeneity. Old-age-specific risk shock is a shock that affects the income risk structure of each age group in a quite complicated way. First, income risk is autocorrelated, so the effect of this shock on the old-age income risk gradually fades over time. Second, the exposure to the shock depends on age, with older workers being more exposed to the shock. Finally, age groups differ by the time duration of their exposure to risky labor income, with older workers facing fewer risky periods ahead than younger workers. Now, we isolate the role of income risk heterogeneity in shaping the age profile of consumption response to the old-age-specific risk shock. Assuming constant labor income MPC and prudence across households set at the mean values and denoted by μ and ν respectively, we obtain a corresponding counterfactual response from Corollary 1 as $S_1 = -\mu\nu \int_t^{\bar{t}} e^{-(r+\frac{1}{\bar{t}-t})(h-t)} \Delta\sigma_h^2 dh$ for households of age $g = t - \underline{t}$. Thus, in this counterfactual, the consumption response varies only with the discounted sum of present and expected future risk changes.

Figure 6a ("risk profile effect") plots the result of this exercise. In response to the old-age-specific risk shock, the oldest workers exhibit the largest drop in consumption because they experience the greatest overall increase in expected lifetime risk. We see that the disproportionate effect of our shock on old-age income risk holds not only for contemporaneous risk but also for the expected discounted sum of future risks.

Age heterogeneity in labor income MPC and prudence. While the risk of older workers increases disproportionately after the old-age-specific income risk shock, their sensitivity to risk is generally smaller due to smaller labor income MPC and prudence. In the second counterfactual exercise, we evaluate how the age dimension of labor income MPC and prudence heterogeneity affects the age profile of consumption response; in this exercise, we abstract from MPC/prudence variation across wealth levels within each age group. Specifically, we compute the counterfactual response as $S_2 = -\int_t^{\bar{t}} e^{-(r+\frac{1}{\bar{t}-t})(h-t)} \cdot E_G[\mu_h^i] \cdot E_G[\nu_h^i] \cdot \Delta\sigma_h^2 dh$ where $E_G[\mu_h^i]$ and $E_G[\nu_h^i]$ denote average MPC and prudence within each age group taken across wealth levels. Figure 6a ("age heterogeneity effect") plots the result of this counterfactual. While taking age heterogeneity in sensitivity to risk into account generates a U-shaped profile of consumption response to old-age risk shock, the maximum response is observed at a somewhat older age, compared to our empirical results, meaning that wealth heterogeneity within each age group also plays a role in shaping the empirically plausible response.

Finally, we also compute the age profile of consumption response to a uniform risk shock in Figure 6b. We see that in the absence of labor income MPC and prudence heterogeneity (S1) response to this shock is almost uniform across age groups. Taking into account heterogeneous sensitivity to risk, makes younger workers relatively more sensitive to this shock, in line with our empirical evidence.

5 Conclusions

In this paper, we analyze the age-specific nature of business cycle fluctuations in income risk, and its implications for consumption dynamics across age groups. Specifically, we document that business cycle fluctuations in income risk faced by older workers are more pronounced compared to those faced by other age groups. Then we estimate the two types of income risk shocks: the old-age-specific and the uniform.

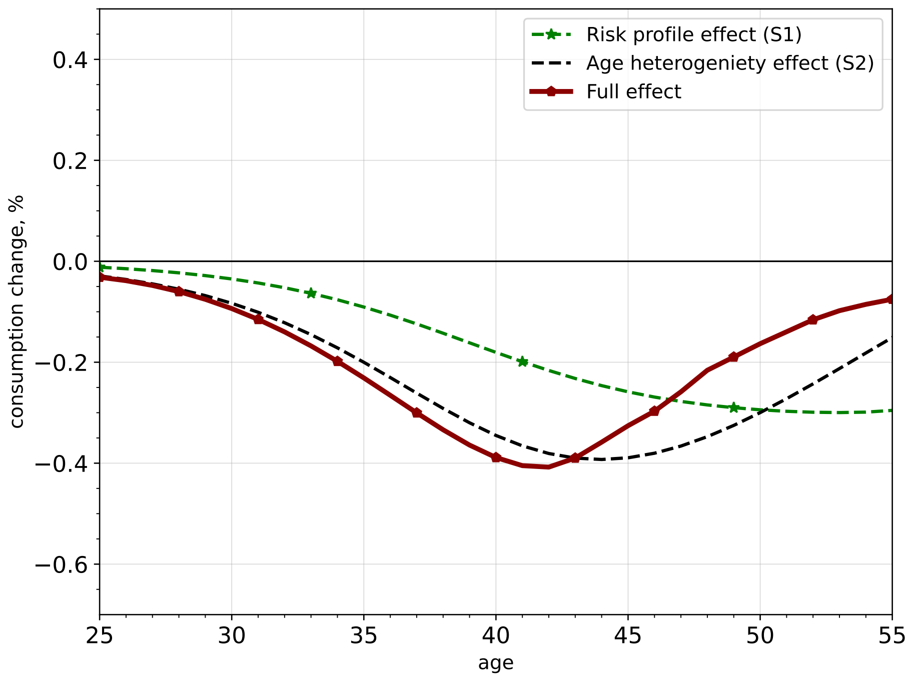
The old-age-specific income risk shocks disproportionately affect the income risk of older workers. Yet, older workers' consumption does not exhibit a strong response to this shock. In contrast, middle-aged workers show the largest consumption drop in response to old-age-specific income risk shock. This evidence suggests that the old-age-specific income risk shock contains information about the future possible income risk for middle-aged workers.

Then we evaluate the ability of a standard life-cycle consumption model featuring rational forward-looking individuals with the precautionary saving motive to account for our empirical finding: the U-shaped age profile of consumption response to the old-age-specific risk shock. We demonstrate that a Huggett-type OLG model can replicate this empirical profile, provided that realistic heterogeneity in labor income MPC and prudence are taken into account.

Our findings have notable policy implications. Policymakers aiming to stabilize consumption over the life cycle could benefit from age-targeted interventions, particularly those designed to reduce income volatility for older workers. For instance, enhancing job stability for older workers or strengthening age-specific social safety

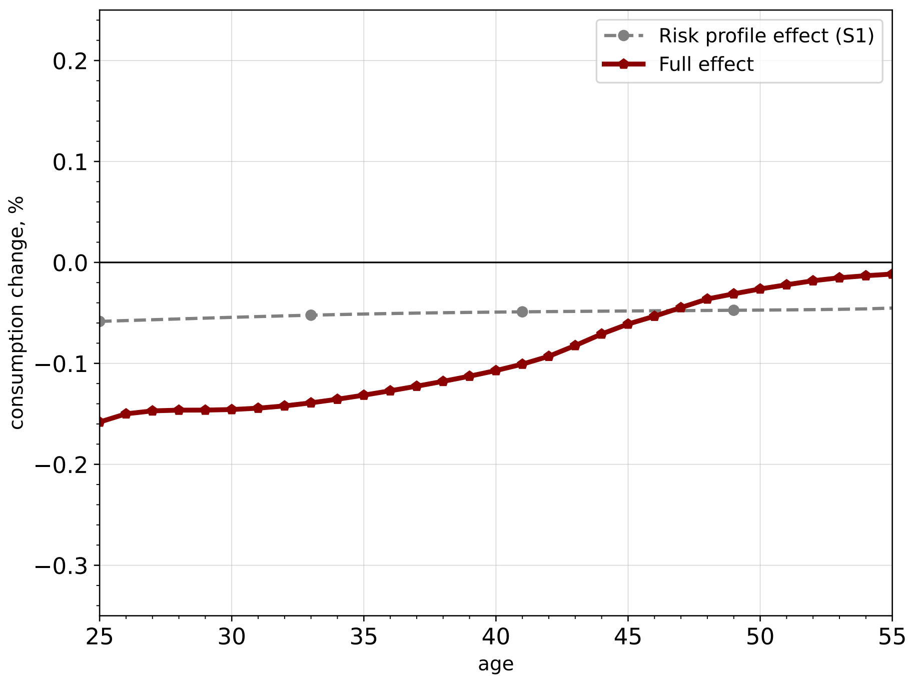
Figure 6: Model-based age profile of consumption response to shocks

(a) Old-age income risk shock



Age profile of the model consumption response cumulative over 6 quarters, normalized by the number of quarters.

(b) Uniform income risk shock



Age profile of the model consumption response cumulative over 6 quarters, normalized by the number of quarters.

nets could not only mitigate the direct effects of income risk on older individuals but also provide a sense of future security for younger cohorts. As middle-aged individuals appear to respond strongly to the risks they foresee facing in their own later years, such interventions may reduce precautionary savings needs across the population, potentially boosting aggregate demand. Additionally, retirement policy reforms that consider income risk stabilization for those nearing retirement could foster a more resilient economy by alleviating consumption fluctuations across multiple age groups.

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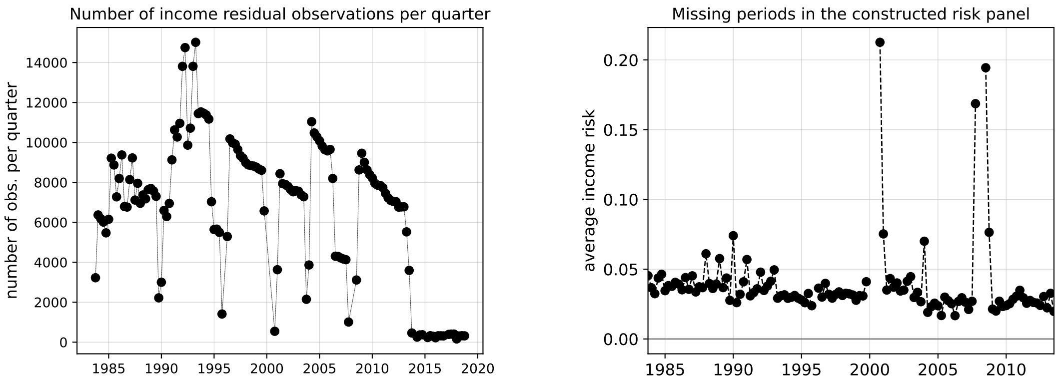
Appendices

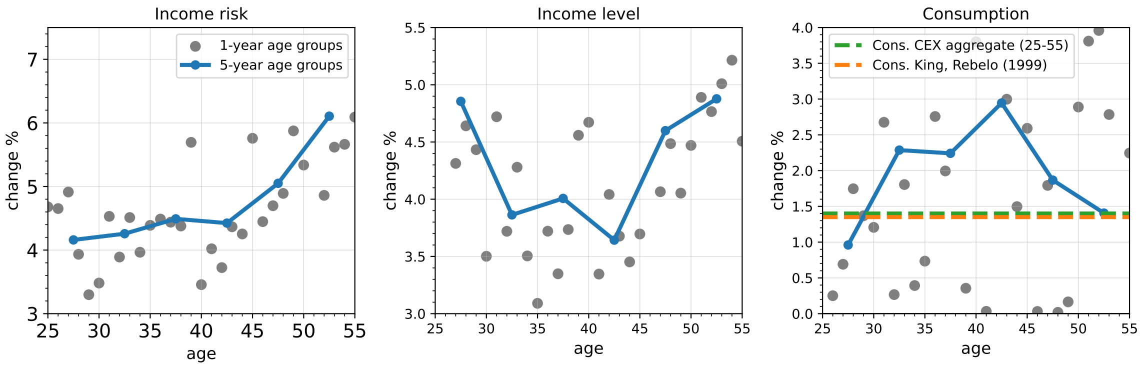
A Empirical Appendix

Figure A.1: Data availability and outliers



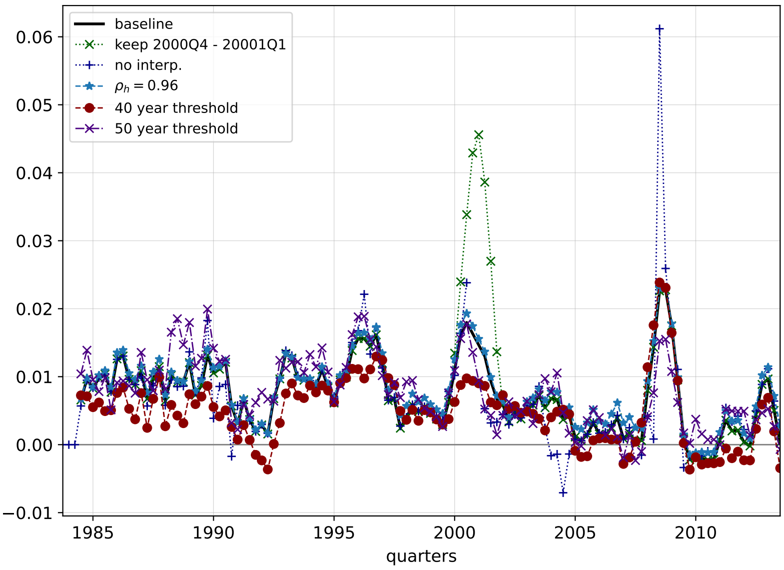
Left panel: the number of available observations by quarter (starting from the 2014 SIPP survey, there are fewer observations due to changes in survey design—see the 2014 SIPP User's Guide). Right panel: average income risk across age groups for each quarter. There are three dates for which there are not enough observations to compute moments. These dates correspond to transitions from the 1993 to 1996 panel, from the 1996 to 2001 panel, and from the 2004 to 2008 panel. The observations for 2000Q4 and 2001Q1 may represent outliers and are therefore set to missing in the baseline estimation..

Figure A.2: Age profiles of business cycle volatility



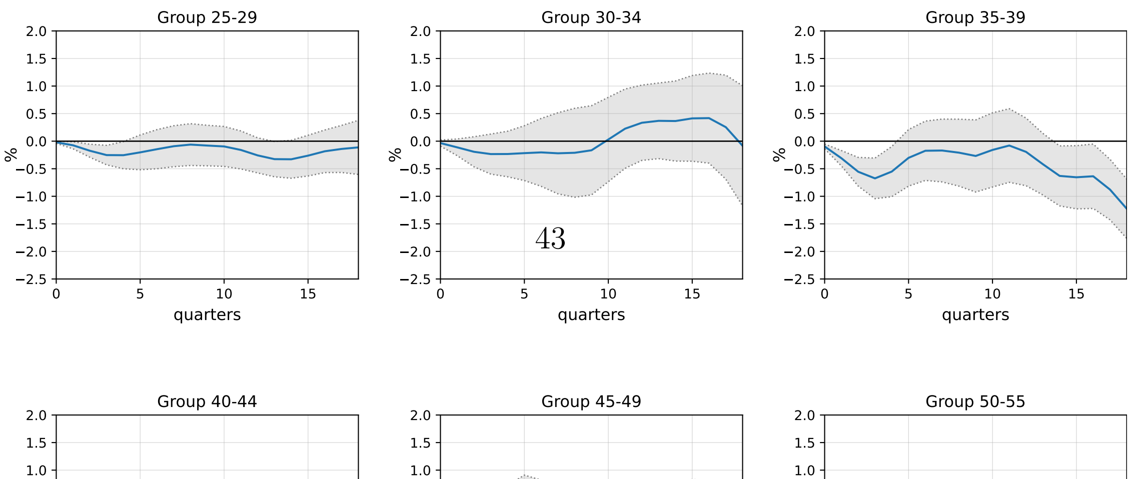
This figure plots age profiles of business cycle volatility of income risk (left panel), income level (middle panel), and consumption (right panel). The right panel also plots the aggregate consumption volatility from CEX and the corresponding measure from King and Rebelo (1999).

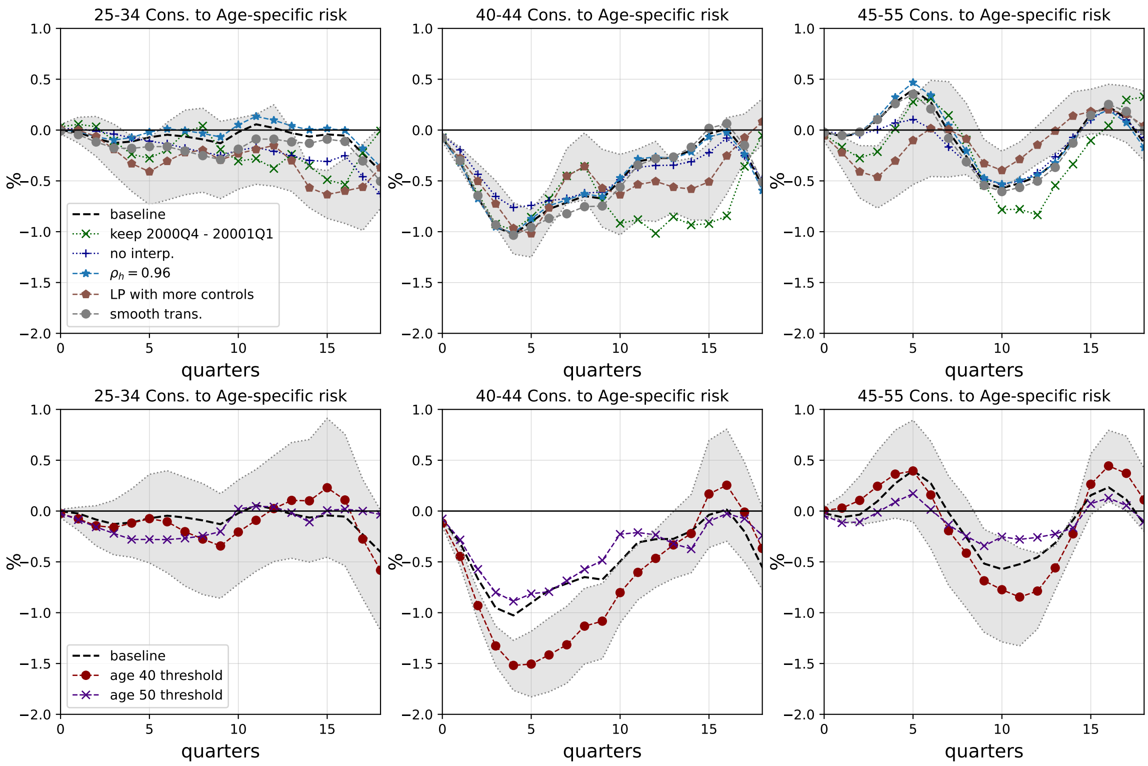
Figure A.3: Age-specific risk over time: robustness checks



The figure plots age-specific income risk over time in the baseline estimation and for the robustness checks.

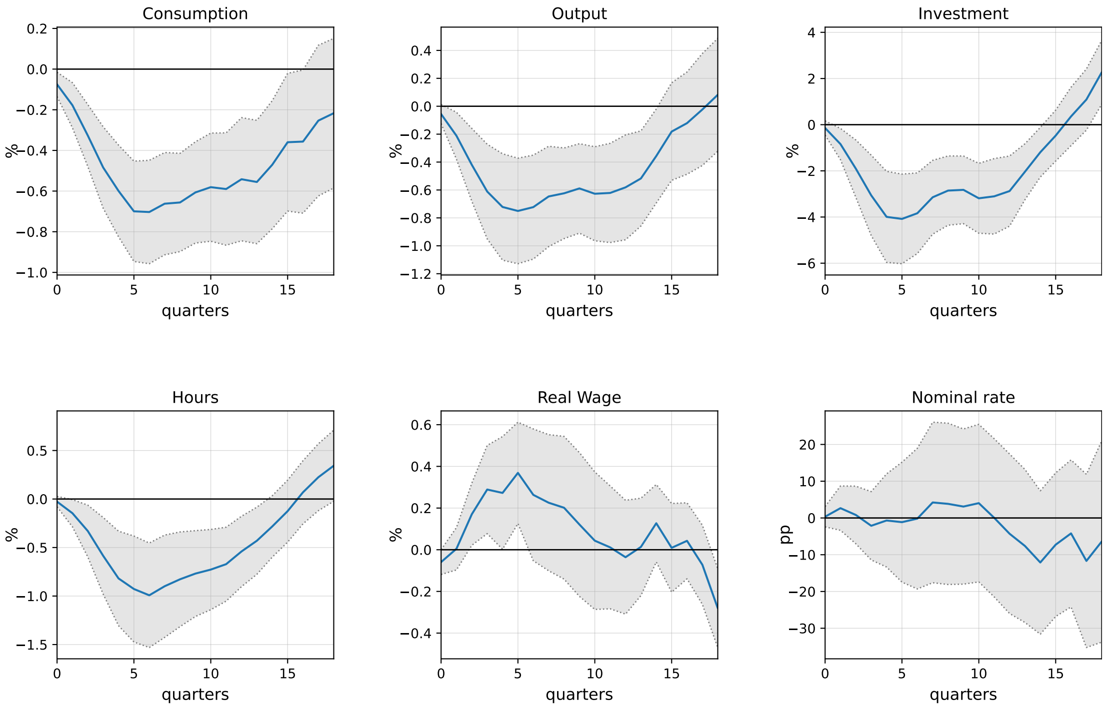
Figure A.4: Consumption response to Age-specific risk shock for six disaggregated age groups





This figure shows the impulse response function (IRF) of consumption to age-specific risk shocks computed for both the baseline response and the robustness checks. Confidence bands are at the 66% level.

Figure A.5: Aggregate response to Age-specific income risk shock



This figure shows the aggregate variables' response to age-specific risk shocks for six disaggregated age groups. Confidence bands are 66%. All variables are retrieved from FRED (PCECC96, GDPC1, GPDIC1, HOANBS, LES1252881600Q, TB3MS).

B Model Appendix

B.1 Proofs

Proof of Proposition 1 (Consumption function). First order conditions are $u'_t = u'(c_t) = \lambda_t$, $E_t\dot{\lambda}_t/\lambda_t = r - \rho_t$, which yields Euler equation $E_t du'_t = u' \cdot (r - \rho_t)dt$.

Following Caballero (1990), we guess the law of motion of consumption as $dc_t = \Gamma_t dt + dv_t$, where dv_t is an innovation at date t. Substituting this law of motion into the Euler equation and performing a second-order Taylor expansion around c_t we obtain

$$\frac{1}{2}u'''(c_t) \cdot Var(dv_t) + u''(c_t)\Gamma_t dt = u'(c_t) \cdot (r - \rho_t)dt$$

Let consumption be related to permanent labor income as $c_t = \mu_t y_t + c^a_t$, where μ_t is MPC out of permanent labor income, and c^a_t collect all determinants of consumption independent of labor income. Then $Var_t(dv_t) = Var_t(dc_t) = \mu^2_t Var_t(dy_t) = \mu^2_t \sigma^2_t dt$ and Γ_t is

$$\Gamma_t = -\frac{1}{2} \cdot \frac{u'''(c_t)}{u''(c_t)} \cdot \mu^2_t \sigma^2_t + \frac{u'(c_t)}{u''(c_t)}(r - \rho_t) = \frac{1}{2} \cdot \eta_t \mu^2_t \sigma^2_t - I_t$$

where $\eta_t = -\frac{u'''(c_t)}{u''(c_t)}$ is the coefficient of prudence for the utility function, and $I_t = -\frac{u'(c_t)}{u''(c_t)}(r - \rho_t)$ is the impatience term.

The expected consumption at date h is $E_t c_h = c_t + \int_t^h E_t \Gamma_h dh$. The intertemporal budget constraint is

$$E_t \int_t^{\bar{t}} e^{-r(h-t)} c_h dh = a_t + E_t \int_t^{\bar{t}} e^{-r(h-t)} y_h dh \equiv L_t$$

where L_t is the expected lifetime wealth.

Let $X_t = \int_t^{\bar{t}} e^{-r(h-t)} dh$ and $x_h = e^{-r(h-t)}$. The left hand side of the intertemporal budget constraint is $X_t c_t + \int_t^{\bar{t}} x_h \left[\int_t^h \Gamma_s ds\right] dh$.

Now, we compute $\int_t^{\bar{t}} x_h \left[\int_t^h \Gamma_s ds\right] dh$. Let $G(h) = \int_t^h \Gamma_s ds$, $G(t) = 0$ and let

$F(h) = \int_t^h x_s ds$, $F(t) = 0$. Let also $F'(h) = f(h) = x_h$ and $G'(h) = g(h) = \Gamma_h$. Substituting these definitions into the integral, we obtain

$$\int_t^{\bar{t}} g(h) f(h) \left[\int_h^{\bar{t}} \frac{f(s)}{f(h)} ds \right] dh$$

Since $\int_h^{\bar{t}} \frac{f(s)}{f(h)} ds = \int_h^{\bar{t}} e^{-\rho_t(s-h)} ds = X_h$, we obtain the value of integral

$$\int_t^{\bar{t}} \Gamma_h x_h X_h dh$$

The intertemporal budget constraint is then

$$X_t c_t + E_t \int_t^{\bar{t}} \Gamma_h x_h X_h dh = L_t$$

Substituting for Γ_t and x_t we obtain

$$X_t c_t + \frac{1}{2} \cdot E_t \int_t^{\bar{t}} e^{-r(h-t)} X_h \eta_h \mu_h^2 \sigma_h^2 dh - E_t \int_t^{\bar{t}} e^{-r(h-t)} X_h I_h = L_t$$

Now we express this equation in terms of labor income risk prudence ν_t. When labor income risk at time t increases by $\Delta\sigma_t^2$, the corresponding decrease in consumption is $\Delta c_t = -\frac{1}{2}\eta_t \mu_t^2 \Delta\sigma_t^2$. At the same time, to maintain unchanged consumption, the compensating increase in labor income is $\tilde{y}_t = \nu_t \Delta\sigma_t^2$. Without this transfer, consumption would drop by $\Delta c_t = -\mu_t \tilde{y}_t$. By equating these two expressions for the change in consumption, we derive the expression for intertemporal labor income risk prudence $\nu_t = \frac{1}{2} \cdot \eta_t \mu_t$ for any t. Substituting this into the budget constraint and expressing it in terms of consumption yields the result.

□

Proof of Corrolary 1 (Age-specific consumption response to risk shock). In the neighborhood of the zero-risk steady state, the first-order response of individual consumption to risk change $\Delta\sigma_h^2$

$$\Delta c_t^i \approx -\int_t^{\bar{t}} e^{-r(h-t)} \cdot \frac{X_h}{X_t} \cdot (\mu_h^i \nu_h^i [E_t \Delta\sigma_h^2]) dh$$

Moreover, for small r, we have $\frac{X_h}{X_t} = \frac{\int_h^{\bar{t}} e^{-r(s-h)} ds}{\int_t^{\bar{t}} e^{-r(s-t)} ds} = \frac{1-e^{-r(\bar{t}-h)}}{1-e^{-r(\bar{t}-t)}} \approx \frac{\bar{t}-h}{\bar{t}-t} = 1 - \frac{h-t}{\bar{t}-t} \approx e^{-\frac{h-t}{\bar{t}-t}}$. Hence, the household's consumption response to risk change $\Delta\sigma_h^2$ is

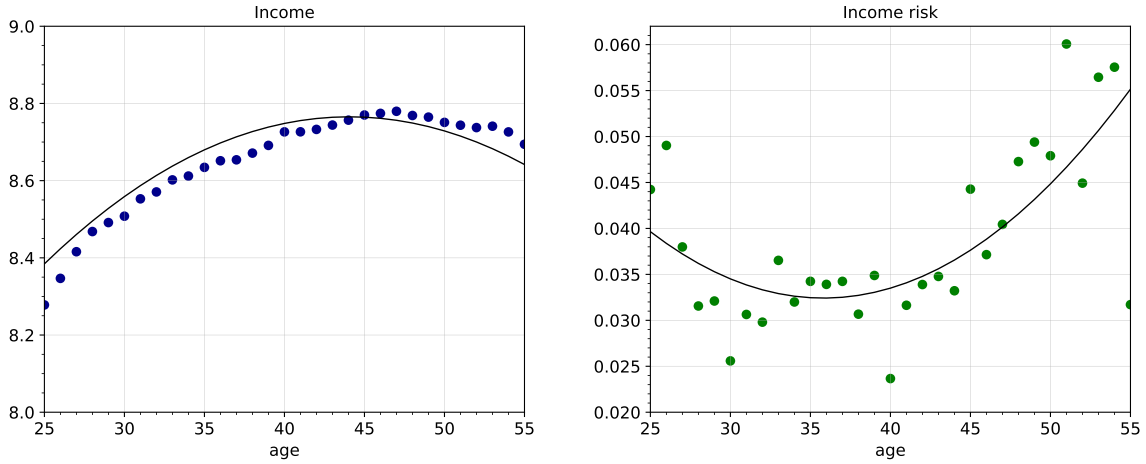
$$\Delta c_t^i \approx -\int_t^{\bar{t}} e^{-(r+\frac{1}{\bar{t}-t})(h-t)} \cdot \mu_h^i \nu_h^i \cdot \Delta\sigma_h^2 dh$$

To obtain the age-specific response, we integrate this expression across all members of the group, which yields the result.

□

B.2 Model calibration

Figure B.6: Stationary age income and risk: data and model



The left panel plots the estimated quarterly deterministic after-tax income profile in logarithms. The right panel depicts the estimated quarterly labor income variance for each age.

B.3 HJB equation

Recall the sequential form of the household problem

$$V(a,z,h,t) = \max_{c_h} E_t \left\{ \int_t^{\underline{t}+\bar{g}} \beta_h \cdot u(c_h) dh + \beta^T \cdot U^T(a_{\underline{t}+\bar{g}}) \right\}$$

$$s.t.\quad \dot{a}_h = r \cdot a_h - c_h + I\{g < R\} \cdot y_h + I\{g \geq R\} \cdot b$$

$$y_t = \bar{y} \cdot e^{\chi_g z_t}$$

$$dz_t = -\phi z_t dt + \sigma_g dW_t$$

$$a_h \geq -d$$

Define $V(a,z,h,t) = V(a,z,h) \equiv V_{a,z,h}$. The HJB equation:

$$(1-\beta_h)V_{a,z,h} = \max_c u(c) + \frac{\partial V_{a,z,h}}{\partial a}\dot{a} + \frac{\partial V_{a,z,h}}{\partial z}\mu_z + \frac{\partial^2 V_{a,z,h}}{\partial z^2}\frac{\sigma_z^2}{2} + \frac{\partial V_{a,z,h}}{\partial t}$$

Terminal condition:

$$V_{a,z,\bar{g}} = V_{term}$$

$$V_{term} = I\{a \geq 0\} \cdot V_{pos} + I\{a < 0\} \cdot V_{neg}$$

$$V_{pos} = \frac{(a+\phi_3)^{(1-\sigma)}}{1-\sigma} + \phi_4$$

$$V_{neg} = -\phi_2(a-\phi_1)^2 + \phi_2 * \phi_1^2$$

$$\phi_3 = (2\phi_1\phi_2)^{-\frac{1}{\sigma}}$$

$$\phi_4 = -\frac{\phi_3^{(1-\sigma)}}{1-\sigma}$$

Two parameters ϕ_1, ϕ_2 govern the strength and the curvature of the bequest motive.